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MECHANICAL ENGINEERING PROGRAM
SCHOOL OF MECHATRONIC ENGINEERING
UniMAP

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Outline

- TYPE OF GEARS
- SPURS AND HELICAL GEARS
- INTERFERENCE
- THE LEWIS BENDING EQUATION
Gear

CO1:

ABILITY TO DESIGN OF MECHANICAL COMPONENTS IN MECHANICAL SYSTEMS
A gear is a component within a transmission device that transmits rotational force to another gear or device.

A gear is different from a pulley in that a gear is a round wheel that has linkages ("teeth" or "cogs") that mesh with other gear teeth, allowing force to be fully transferred without slippage.

Depending on their construction and arrangement, geared devices can transmit forces at different speeds, torques, or in a different direction, from the power source.
Gear

The most common situation is for a gear to mesh with another gear, but a gear can mesh with any device having compatible teeth, such as linear moving racks.

The gear's most important feature is that gears of unequal sizes (diameters) can be combined to produce a mechanical advantage, so that the rotational speed and torque of the second gear are different from those of the first. In the context of a particular machine, the term "gear" also refers to one particular arrangement of gears among other arrangements (such as "first gear"). Such arrangements are often given as a ratio, using the number of teeth or gear diameter as units.
Gear

► Types of Gears
1. Spur gear
2. Helical Gear
3. Bevel Gear
4. Worm Gear
Spur Gear

Spur gears have teeth parallel to the axis of rotation and are to transmit motion from one shaft to another parallel shaft.
Spur Gear

(a) Spur gear with spoked design

(b) Spur gear with solid hub
Spur gear

- Pair of spur gears.
- The pinion drives the gear.
Helical gears have teeth inclined to the axis of rotation.
Bevel gear

Bevel gears have teeth formed on conical surfaces and are used mostly for transmitting motion between intersecting shafts.
Worm gear

The direction of rotation of the worm gear, also called the worm wheel, depends upon the direction of rotation of the worm and upon whether the worm teeth are cut right-hand or left-hand.
A rack is a toothed bar or rod that can be thought of as a sector gear with an infinitely large radius of curvature. Torque can be converted to linear force by meshing a rack with a pinion: the pinion turns; the rack moves in a straight line.

Racks also feature in the theory of gear geometry, where, for instance, the tooth shape of an interchangeable set of gears may be specified for the rack (infinite radius), and the tooth shapes for gears of particular actual radii then derived from that.
Nomenclature

- Face width
- Top land
- Addendum circle
- Addendum
- Circular pitch
- Tooth thickness
- Width of space
- Pitch circle
- Face
- Flank
- Bottom land
- Clearance circle
- Dedendum
- Clearance
- Fillet radius
- Dedendum circle
**Nomenclature**

- **Pitch circle** is a theoretical circle upon which all calculations are usually based; its diameter is the *pitch diameter*.
- A *pinion* is the smaller of two mating gears.
- The larger is often called *gear*.
- The *circular pitch* \((p)\) is the distance, measured on the pitch circle, from a point on one tooth to a corresponding point on an adjacent tooth.
- The circular pitch is equal to the sum of the *tooth thickness* and the *width of space*. 
Nomenclature

- **The module** \( (m) \) is the ratio of the pitch diameter to the number of teeth.
- **The addendum** \( (a) \) is the radial distance between the top land and the pitch circle.
- **The dedendum** \( (b) \) is the radial distance from bottom land and the dedendum.
- **The whole depth** \( (h) \) is the sum of the addendum and the dedendum.
\[ D_o = \text{outside diameter} = D + 2a \]
\[ D_R = \text{root diamter} = D - 2b \]

\[ h_t = \text{whole depth} = a + b = h_k + c \]
\[ h_k = \text{working depth} = a + a = 2a \]

\[ c = \text{clearance} = c = b - a \]

\[ t = \text{tooth thickness} = \frac{p}{2} = \frac{\pi}{2P} \]
The **clearance circle** is a circular that is tangent to addendum circle of mating gear.

The **clearance** \((c)\) is gear exceeding the sum of addendum and the dedendum.

The **backlash** is the amount by which the width of a tooth space exceeds the thickness of the engaging tooth measured on the pitch circles.
Fundamental

- $P =$ diametral pitch, teeth per inch
- $m =$ module, mm.
- $d =$ pitch diameter, mm
- $N =$ number of teeth.
- $p =$circular pitch

\[ P = \frac{N}{d} \]
\[ m = \frac{d}{N} \]

$$pP = \pi$$

\[ p = \frac{\pi d}{N} = \pi m \]

\[ m = \frac{1}{P \text{ in}} \]
\[ m = \frac{25.4}{P \text{ mm}} \]
# Standard modules

<table>
<thead>
<tr>
<th>Module (mm)</th>
<th>Equivalent $P_d$</th>
<th>Closest standard $P_d$ (teeth/in)</th>
<th>Metric module $m = 25.4/P_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>84.667</td>
<td>80</td>
<td>0.31</td>
</tr>
<tr>
<td>0.4</td>
<td>63.500</td>
<td>64</td>
<td>0.39</td>
</tr>
<tr>
<td>0.5</td>
<td>50.800</td>
<td>48</td>
<td>0.52</td>
</tr>
<tr>
<td>0.8</td>
<td>31.750</td>
<td>32</td>
<td>0.79</td>
</tr>
<tr>
<td>1</td>
<td>25.400</td>
<td>24</td>
<td>1.05</td>
</tr>
<tr>
<td>1.25</td>
<td>20.320</td>
<td>20</td>
<td>1.27</td>
</tr>
<tr>
<td>1.5</td>
<td>16.933</td>
<td>16</td>
<td>1.58</td>
</tr>
<tr>
<td>2</td>
<td>12.700</td>
<td>12</td>
<td>2.11</td>
</tr>
<tr>
<td>2.5</td>
<td>10.160</td>
<td>10</td>
<td>2.54</td>
</tr>
<tr>
<td>3</td>
<td>8.466</td>
<td>8</td>
<td>3.17</td>
</tr>
<tr>
<td>4</td>
<td>6.350</td>
<td>6</td>
<td>4.23</td>
</tr>
<tr>
<td>5</td>
<td>5.080</td>
<td>5</td>
<td>5.08</td>
</tr>
<tr>
<td>6</td>
<td>4.233</td>
<td>4</td>
<td>6.35</td>
</tr>
<tr>
<td>8</td>
<td>3.175</td>
<td>3</td>
<td>8.46</td>
</tr>
<tr>
<td>10</td>
<td>2.540</td>
<td>2.5</td>
<td>10.16</td>
</tr>
<tr>
<td>12</td>
<td>2.117</td>
<td>2</td>
<td>12.7</td>
</tr>
<tr>
<td>16</td>
<td>1.587</td>
<td>1.5</td>
<td>16.9</td>
</tr>
<tr>
<td>20</td>
<td>1.270</td>
<td>1.25</td>
<td>20.3</td>
</tr>
<tr>
<td>25</td>
<td>1.016</td>
<td>1</td>
<td>25.4</td>
</tr>
</tbody>
</table>
Gear-tooth size

[Diagram of gear tooth sizes]
Tooth systems

A tooth system is a standard that specifies the relationships involving addendum, dedendum, working depth, tooth thickness and pressure angle.

Table 13-1

<table>
<thead>
<tr>
<th>Tooth System</th>
<th>Pressure Angle $\phi$, deg</th>
<th>Addendum $a$</th>
<th>Dedendum $b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full depth</td>
<td>20</td>
<td>$1/P_d$ or $1m$</td>
<td>$1.25/P_d$ or $1.25m$</td>
</tr>
<tr>
<td>$22\frac{1}{2}$</td>
<td>20</td>
<td>$1/P_d$ or $1m$</td>
<td>$1.25/P_d$ or $1.25m$</td>
</tr>
<tr>
<td>25</td>
<td>20</td>
<td>$1/P_d$ or $1m$</td>
<td>$1.35/P_d$ or $1.35m$</td>
</tr>
<tr>
<td>20</td>
<td>20</td>
<td>0.8/$P_d$ or 0.8$m$</td>
<td>$1/P_d$ or 1$m$</td>
</tr>
</tbody>
</table>

3Standardized by the American Gear Manufacturers Association (AGMA). Write AGMA for a complete list of standards, because changes are made from time to time. The address is: 1500 King Street, Suite 201, Alexandria, VA 22314; or wwwAGMA.org.
Diametral pitch

Table 13-2

Tooth Sizes in General Uses

<table>
<thead>
<tr>
<th>Diametral Pitch</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coarse</td>
</tr>
<tr>
<td>Fine</td>
</tr>
</tbody>
</table>

- Coarse: 2, 2\(\frac{1}{4}\), 2\(\frac{1}{2}\), 3, 4, 6, 8, 10, 12, 16
- Fine: 20, 24, 32, 40, 48, 64, 80, 96, 120, 150, 200

Table 13-3

Tooth Proportions for 20° Straight Bevel-Gear Teeth

<table>
<thead>
<tr>
<th>Item</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Working depth</td>
<td>(h_1 = \frac{2.0}{P})</td>
</tr>
<tr>
<td>Clearance</td>
<td>(c = (0.188/P) + 0.002) in</td>
</tr>
<tr>
<td>Addendum of gear</td>
<td>(a_G = \frac{0.54}{P} + \frac{0.460}{P(m_{20})^2})</td>
</tr>
<tr>
<td>Gear ratio</td>
<td>(m_G = N_G/N_P)</td>
</tr>
<tr>
<td>Equivalent 90° ratio</td>
<td>(m_{20} = m_G) when (\Gamma = 90°)</td>
</tr>
<tr>
<td></td>
<td>(m_{20} = m_G \cdot \sqrt{\frac{\cos \gamma}{\cos \Gamma}}) when (\Gamma \neq 90°)</td>
</tr>
<tr>
<td>Face width</td>
<td>(F = 0.3A_0) or (F = \frac{10}{P}), whichever is smaller</td>
</tr>
<tr>
<td>Minimum number of teeth</td>
<td>Pinion</td>
</tr>
<tr>
<td></td>
<td>Gear</td>
</tr>
</tbody>
</table>
# Standard tooth for helical gears

<table>
<thead>
<tr>
<th>Quantity*</th>
<th>Formula</th>
<th>Quantity*</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Addendum</td>
<td>( \frac{1.00}{P_n} )</td>
<td>External gears:</td>
<td></td>
</tr>
<tr>
<td>Dedendum</td>
<td>( \frac{1.25}{P_n} )</td>
<td>Standard center distance</td>
<td>( \frac{D + d}{2} )</td>
</tr>
<tr>
<td>Pinion pitch diameter</td>
<td>( \frac{N_p}{P_n \cos \psi} )</td>
<td>Gear outside diameter</td>
<td>( D + 2a )</td>
</tr>
<tr>
<td>Gear pitch diameter</td>
<td>( \frac{N_G}{P_n \cos \psi} )</td>
<td>Pinion outside diameter</td>
<td>( d + 2a )</td>
</tr>
<tr>
<td>Normal arc tooth thickness→</td>
<td>( \frac{\pi}{P_n} - \frac{B_n}{2} )</td>
<td>Gear root diameter</td>
<td>( D - 2b )</td>
</tr>
<tr>
<td>Pinion base diameter</td>
<td>( d \cos \phi_t )</td>
<td>Pinion root diameter</td>
<td>( d - 2b )</td>
</tr>
<tr>
<td>Gear base diameter</td>
<td>( D \cos \phi_t )</td>
<td>Internal gears:</td>
<td></td>
</tr>
<tr>
<td>Base helix angle</td>
<td>( \tan^{-1}(\tan \psi \cos \phi_t) )</td>
<td>Center distance</td>
<td>( \frac{D - d}{2} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Inside diameter</td>
<td>( D - 2a )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Root diameter</td>
<td>( D + 2b )</td>
</tr>
</tbody>
</table>

*All dimensions are in inches, and angles are in degrees.

†\( B_n \) is the normal backlash.
Problem

Problem 13-1

A 17-tooth spur pinion has a diametral pitch of 8 teeth/in, runs at 1120 rev/min, and drives a gear at a speed of 544 rev/min.

Find the number of teeth on the gear and the theoretical center-to-center distance.

\[ d_p = \frac{N_p}{P} = \frac{17}{8} = 2.125\text{in} \]
\[ d_G = \frac{n_2}{n_3} d_p = \frac{1120}{544} (2.125\text{in}) = 4.375\text{in} \]
\[ N_G = P d_G = 8(4.375) = 35\text{teeth} \]
\[ C = \frac{(2.125+4.375)}{2} = 3.25\text{in} \]

Problem 13-2

A 15-tooth spur pinion has a module of 3mm and runs at a speed of 1600 rev/min. The driven gear has 60 teeth. Find the speed of the driven gear, the circular pitch, and the theoretical center-to-center distance.

Solution

\[ n_G = 1600 \left(\frac{15}{60}\right) = 400\text{rev/min} \]
\[ p = \pi m = 3\pi\text{mm} \]
\[ C = \frac{[3(15+60)]}{2} = 112.5\text{mm} \]
The pitch-line velocity

\[ V = \left| r_1 \omega_1 \right| = \left| r_2 \omega_2 \right| \]

\[ \frac{\omega_1}{\omega_2} = \frac{r_2}{r_1} \]
Problem 13-3

A spur gearset has a module of 4mm and a velocity ratio of 2.80. The pinion has 20 teeth.

Find the number of teeth on the driven gear, the pitch diameter, and the theoretical center-to-center distance.

Solution

\[ N_G = N_P (2.80) = 20(2.80) = 56 \text{ teeth} \]
\[ d_G = N_G m = (56)(4) = 224 \text{ mm} \]
\[ d_P = N_P m = (20)(4) = 80 \text{ mm} \]
\[ C = \frac{d_P + d_G}{2} = \frac{80 + 224}{2} = 152 \text{ mm} \]
Suppose we specify that an 18-tooth pinion is to mesh with a 30-tooth gear and that the diametral pitch of the gear set is to be 2 teeth per inch.

The pitch diameters of the pinion and gear are

\[ d_1 = \frac{N_1}{P} = \frac{18}{2} = 9\text{in} \]
\[ r_1 = 4.5\text{in} \]
\[ d_2 = \frac{N_2}{P} = \frac{30}{2} = 15\text{in} \]
\[ r_2 = 7.5\text{in} \]

The center distance is the sum of the pitch radii,

\[ r_1 + r_2 = 4.5\text{in} + 7.5\text{in} = 12\text{in} \]
The construct the pitch circles of radii $r_1$ and $r_2$. These are tangent at P, the pitch point.

Next draw line $ab$, common tangent, through the pitch point.

We designate gear 1 as the driver (CCW).

We draw a line $cd$ through point P at an angle $\phi$ to the common tangent $ab$.

The line $cd$ has three names; namely the pressure line, the generating line, and the line of action.
Relating base circle to the pressure angle $\phi$

\[ r_b = r \cos \phi \]
Pressure angle $\phi$

$\phi = 14\frac{1}{2}^\circ$

$\phi = 20^\circ$

$\phi = 25^\circ$
Example 13-1

A gearset consists of a 16-tooth pinion driving a 40-tooth gear. The diametral pitch \( P \) is 2, and the addendum and dedendum are \( 1/P \) and \( 1.25/P \), respectively. The gears are cut using a pressure angle of \( \phi = 20^\circ \).

Compute the circular pitch, the center distance, and the radii of the base circles.

In mounting these gears, the center distance was incorrectly made \( 1/4 \) in larger.

Compute the new values of the pressure angle and the pitch-circle diameters.

Solution

The circular pitch

The pitch diameters of the pinion \( d_p \) and gear \( d_G \) are, respectively,

\[
\begin{align*}
    d_p &= \frac{N}{P} = \frac{16}{2} = 8\text{in} \\
    d_G &= \frac{N}{P} = \frac{40}{2} = 20\text{in}
\end{align*}
\]

The center distance

\[
c = \frac{d_p + d_G}{2} = \frac{8 + 20}{2} = 14\text{in}
\]
Pressure angle $20^\circ$, the base-circle radii is

\[
r_B = r \cos \phi
\]
\[
r_B (\text{pinion}) = \frac{8}{2} \cos 20^\circ = 3.76 \text{in}
\]
\[
r_B (\text{gear}) = \frac{20}{2} \cos 20^\circ = 9.40 \text{in}
\]

Designating $d_p'$ and $d_G'$ as the new pitch-circle diameters, the $\frac{1}{4}$ in increase in the center distance requires that

\[
\frac{d_p' + d_G'}{2} = 14.25 \text{in}
\]

The velocity ratio does not change,

\[
\frac{0.4d_G' + d_G'}{2} = 14.25
\]
\[
1.4d_G' = 28.5 \Rightarrow d_G' = 20.357 \text{in}
\]
\[
d_p' = 0.4(20.357) = 8.143 \text{in}
\]

The new pressure angle $\Phi$

\[
r_b = r \cos \phi
\]
\[
\phi' = \cos^{-1} \frac{r_b (\text{pinion})}{d_p'/2} = \cos^{-1} \frac{3.76}{8.143/2} = 22.56^\circ
\]
Generation of an involute curve

[Diagram showing the generation of an involute curve]

- Base circle
- Pitch circle
- Points A, B, C, D, E, F
- O, O1, O2
Construction of an involute
The involute gear profile is the most commonly used system for gearing today. In an involute gear, the profiles of the teeth are involutes of a circle.

The involute of a circle is the spiraling curve traced by the end of an imaginary taut string unwinding itself from that stationary circle.
Equations of an involute of a parametrical define curve are:

\[
X[x, y] = x - \frac{a}{\sqrt{x'^2 + y'^2}} \int_{t}^{t} \sqrt{x'^2 + y'^2} \, dt
\]

\[
Y[x, y] = y - \frac{a}{\sqrt{x'^2 + y'^2}} \int_{t}^{t} \sqrt{x'^2 + y'^2} \, dt
\]
Tooth contact nomenclature
Point of contact

Path of Action, ANSI/AGMA 1012-G05

A point of contact is any point at which two tooth profiles touch each other.
Surface of action

- The surface of action is the imaginary surface in which contact occurs between two engaging tooth surfaces. It is the summation of the paths of action in all sections of the engaging teeth.

- Plane of Action, ANSI/AGMA 1012-G05

- The plane of action is the surface of action for involute, parallel axis gears with either spur or helical teeth. It is tangent to the base cylinders.
Line of contact

- Line of Contact, ANSI/AGMA 1012-G05
- A line of contact is a line or curve along which two tooth surfaces are tangent to each other.
Zone of action (contact zone)

- **Zone of Action, ANSI/AGMA 1012-G05**

- **Zone of action (contact zone)** for involute, parallel-axis gears with either spur or helical teeth, is the rectangular area in the plane of action bounded by the length of action and the effective **face width**.
Path of contact

Lines of Contact (helical gear), ANSI/AGMA 1012-G05

The path of contact is the curve on either tooth surface along which theoretical single point contact occurs during the engagement of gears with crowned tooth surfaces or gears that normally engage with only single point contact.
Length of Action, 
ANSI/AGMA 1012-G05

Length of action is the distance on the line of action through which the point of contact moves during the action of the tooth profile.
Involute-toothed pinion and rack

\[ p_b = p_c \cos \phi \]

\[ p_b = \text{the base pitch} \]
Internal gear and pinion
Contact ratio \((m_c)\)

\[ m_c = \frac{q_t}{p} \]

\[ q_t = \text{the arc of action} (q_t) \]

\[ m_c = \frac{L_{ab}}{p \cos \phi} \]
Problem 13-4

A 21-tooth spur pinion mates with a 28-tooth gear. The diametral pitch is 3 teeth/in and the pressure angle is 20°. Make a drawing of the gears showing one tooth on each gear.

Find the addendum (a), dedendum (b), clearance (c), circular pitch (p), tooth thickness (t), and base-circle diameters; the lengths of the arc approach, recess, and action; and the base pitch and contact ratio.
\[a = \frac{1}{P} = \frac{1}{3} = 0.333\text{in}\]

\[b = \frac{1.25}{P} = \frac{1.25}{3} = 0.4167\text{in}\]

\[c = b - a = 0.4167 - 0.3333 = 0.0834\text{in}\]

\[p = \frac{\pi}{P} = \frac{\pi}{3} = 1.047\text{in}\]

\[t = \frac{p}{2} = \frac{1.047}{2} = 0.523\text{in}\]

**Pinion**

\[d_1 = \frac{N_1}{P} = \frac{21}{3} = 7\text{in}\]

\[d_{1b} = d_1 \cos 20^\circ = 7 \cos 20^\circ = 6.578\text{in}\]

\[d_2 = \frac{N_2}{P} = \frac{28}{3} = 9.333\text{in}\]

\[d_{2b} = 9.333 \cos 20^\circ = 8.770\text{in}\]

**Base pitch**

\[p_b = p_c \cos \phi = \left(\frac{\pi}{3}\right) \cos 20^\circ = 0.984\text{in}\]

**Contact ratio**

\[m_c = \frac{L_{ab}}{p_b} = \frac{1.53}{0.984} = 1.55\]
The **gear ratio** is the relationship between the number of teeth on two gears that are meshed or two sprockets connected with a common roller chain, or the circumferences of two pulleys connected with a drive belt.

In the picture to the right, the smaller gear (known as the **pinion**) has 13 teeth, while the second, larger gear (known as the **idler** gear) has 21 teeth. The gear ratio is therefore 13/21 or 1/1.62 (also written as 1:1.62).

This means that for every one revolution of the pinion, the gear has made 1/1.62, or 0.62, revolutions. In practical terms, the gear turns more slowly. Suppose the largest gear in the picture has 42 teeth, the gear ratio between the second and third gear is thus 21/42 = 1/2, and for every revolution of the smallest gear the largest gear has only turned 0.62/2 = 0.31 revolution, a total reduction of about 1:3.23.
Gear ratio

- 1st gear 2.97:1
- 2nd gear 2.07:1
- 3rd gear 1.43:1
- 4th gear 1.00:1
- 5th gear 0.84:1
- 6th gear 0.56:1
- reverse 3.28:1
The contact portions of tooth profiles that are not conjugate is called interference.

$N_P = \text{the smallest number of teeth on the pinion without interference.}$

- $k=1$ for full depth teeth.
- $k=0.8$ for stub teeth
- $\phi = \text{pressure angle}$

\[
N_P = \frac{2k}{3\sin^2\phi} \left(1 + \sqrt{1 + 3\sin^2\phi}\right)
\]
Interference

For pressure angle a 20°, with k=1

\[ N_p = \frac{2(1)}{3 \sin^2 20} \left(1 + \sqrt{1 + 3 \sin^2 20}\right) \]
\[ N_p = 12.3 = 13\text{ teeth} \]

Thus a 16-tooth pinion will mesh with a 64-tooth gear without interference.

For \( m=4, \phi = 20° \)

\[ N_p = \frac{2(1)}{(1 + 2(4) \sin^2 20°)} \left(4 + \sqrt{4^2 + (1 + 2(4) \sin^2 20°)}\right) \]
\[ N_p = 15.4 = 16\text{ teeth} \]

The largest gear with a specified pinion that is interference-free is

If the mating gear has more teeth than the pinion, that is, \( m_G = N_G / N_p = m \) more than one, then \( N_p \) is

\[ N_p = \frac{2k}{(1 + 2m) \sin^2 \phi} \left(\frac{m}{m^2 + (1 + 2m) \sin^2 \phi}\right) \]

\[ N_G = \frac{N_p^2 \sin^2 \phi - 4k^2}{4k - 2N_p \sin^2 \phi} \]
Interference

- For a 13-tooth pinion with a pressure angle $\phi$ of 20°

$$N_G = \frac{13^2 \sin^2 20° - 4(1)^2}{4(1) - 2(13) \sin^2 20°} = 16.45 = 16\text{teeth}$$

- For a 13-tooth spur pinion, the maximum number of gear teeth possible without interference is 16.

- The smallest spur pinion that will a rack without interference is

$$N_P = \frac{2(k)}{\sin^2 \phi}$$

$$N_P = \frac{2(1)}{\sin^2 20°} = 17.1 = 18\text{teeth}$$

- For a 20° pressure angle full-depth tooth the smallest number of pinion teeth to mesh with a rack is
The pitch angles are defined by the pitch cones meeting at the apex. They are related to the tooth numbers as follows;

\[
\tan \gamma = \frac{N_P}{N_G} \\
\tan \Gamma = \frac{N_G}{N_P}
\]

The virtual number of teeth (N’) and p is the circular pitch.
Helical gear

- The normal circular pitch $p_n$ and is related to the transverse circular pitch $p_t$ as follows:

$$p_n = p_t \cos \psi$$

- The axial pitch $p_x$;

$$p_x = \frac{p_t}{\tan \psi}$$

- The normal diametral pitch

$$P_n = \frac{P_t}{\cos \psi}$$

$$\cos \psi = \frac{\tan \phi_n}{\tan \phi_t}$$
Helical gear

- Figure shows a cylinder cut by an oblique plane $ab$ at an angle to right section. $\psi$

$$\psi = 0^\circ \Rightarrow R = \frac{D}{2}$$

$$\psi = 90^\circ \Rightarrow R = \infty$$

- The virtual number of teeth is related to the actual number:

$$N' = \frac{N}{\cos^3 \psi}$$
# Helical gears

## Table 13-5

<table>
<thead>
<tr>
<th>Recommended Pressure of Angles and Tooth Depths for Worm Gearing</th>
<th>Lead Angle $\lambda$, deg</th>
<th>Pressure Angle $\phi_{nr}$, deg</th>
<th>Addendum $a$</th>
<th>Dedendum $b_G$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-15</td>
<td>14½</td>
<td>$0.3683p_x$</td>
<td>$0.3683p_x$</td>
<td></td>
</tr>
<tr>
<td>15-30</td>
<td>20</td>
<td>$0.3683p_x$</td>
<td>$0.3683p_x$</td>
<td></td>
</tr>
<tr>
<td>30-35</td>
<td>25</td>
<td>$0.2865p_x$</td>
<td>$0.3314p_x$</td>
<td></td>
</tr>
<tr>
<td>35-40</td>
<td>25</td>
<td>$0.2546p_x$</td>
<td>$0.2947p_x$</td>
<td></td>
</tr>
<tr>
<td>40-45</td>
<td>30</td>
<td>$0.2228p_x$</td>
<td>$0.2578p_x$</td>
<td></td>
</tr>
</tbody>
</table>

## Table 13-6

<table>
<thead>
<tr>
<th>Efficiency of Worm Gearsets for $f = 0.05$</th>
<th>Helix Angle $\psi$, deg</th>
<th>Efficiency $\eta$, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>25.2</td>
<td></td>
</tr>
<tr>
<td>2.5</td>
<td>45.7</td>
<td></td>
</tr>
<tr>
<td>5.0</td>
<td>62.0</td>
<td></td>
</tr>
<tr>
<td>7.5</td>
<td>71.3</td>
<td></td>
</tr>
<tr>
<td>10.0</td>
<td>76.6</td>
<td></td>
</tr>
<tr>
<td>15.0</td>
<td>82.7</td>
<td></td>
</tr>
<tr>
<td>20.0</td>
<td>85.9</td>
<td></td>
</tr>
<tr>
<td>22.0</td>
<td>89.1</td>
<td></td>
</tr>
</tbody>
</table>
Straight bevel gear

- The pitch angles are defined by the pitch cones meeting at the apex. They are related to the tooth numbers as follows:

  \[ \tan \gamma = \frac{N_P}{N_G} \]
  \[ \tan \Gamma = \frac{N_G}{N_P} \]

- The virtual number of teeth (\( N' \)) and \( p \) is the circular pitch.

\[ N' = \frac{2\pi r_b}{p} \]
The normal circular pitch $p_n$ and is related to the transverse circular pitch $p_t$ as follows:

$$p_n = p_t \cos \psi$$

The axial pitch $p_x$:

$$p_x = \frac{p_t}{\tan \psi}$$

The normal diametral pitch:

$$P_n = \frac{P_t}{\cos \psi}$$

$$\cos \psi = \frac{\tan \phi_n}{\tan \phi_t}$$
Helical gear

- Figure shows a cylinder cut by an oblique plane $ab$ at an angle to right section.

$\psi = 0^\circ \Rightarrow R = \frac{D}{2}$

$\psi = 90^\circ \Rightarrow R = \infty$

- The virtual number of teeth is related to the actual number:

$$N' = \frac{N}{\cos^3 \psi}$$
Helical gear

The pressure angle $\Phi_t$

$$\phi_i = \tan^{-1}\left(\frac{\tan \phi_n}{\cos \psi}\right)$$

$$N_p = \frac{2k \cos \psi \left(1 + \sqrt{1 + 3 \sin^2 \phi_t}\right)}{3 \sin^2 \phi_t}$$

$\phi_i = 20^\circ, \psi = 30^\circ$

$$\phi_i = \tan^{-1}\left(\frac{\tan 20^\circ}{\cos 30^\circ}\right) = 22.80^\circ$$

$$N_p = \frac{2(1) \cos 30^\circ \left(1 + \sqrt{1 + 3 \sin^2 22.80\circ}\right)}{3 \sin^2 22.80\circ} = 8.48 = 9\text{ teeth}$$

$$m_G = \frac{N_G}{N_p} = m$$

$$N_p = \frac{N_p^2 \sin^2 \phi_i - 4k^2 \cos^2 \psi}{4k \cos \psi - 2N_p \sin^2 \phi_i}$$

$\phi_n = 20^\circ, \psi = 30^\circ, \phi_i = 22.80^\circ$

$$N_G = \frac{9^2 \sin^2 22.80^\circ - 4(1)^2 \cos^2 30^\circ}{4(1) \cos 30^\circ - 2(9) \sin^3 22.80^\circ} = 12.02 = 12$$

$$N_p = \frac{2k \cos \psi}{\sin^2 \phi_i}$$

$\phi_n = 20^\circ, \psi = 30^\circ, \phi_i = 22.80^\circ$

$$N_p = \frac{2(1) \cos 30^\circ}{\sin^2 22.80^\circ} = 11.5 = 12\text{ teeth.}$$
Worm gears

- $d_G =$ the pitch diameter
- $N_G =$ the number of teeth
- $p_t =$ the transverse circular pitch.
- $p_x =$ the axial pitch
- $\lambda =$ the lead angle
- $\Psi =$ the helix angle

$$d_G = \frac{N_G p_t}{\pi}$$
Worm gears

- The pitch diameter of the worm gear should be selected so as to fall into the range:

\[ \frac{C^{0.875}}{3.0} \leq d_w \leq \frac{C^{0.875}}{1.7} \]

- \( C \) = the center distance.
- \( L \) = the lead.
- \( \lambda \) = the lead angle.

\[ \tan \lambda = \frac{L}{\pi d_w} \]
## Worm gears

### Table 13–5

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<tbody>
<tr>
<td>Angles and Tooth</td>
<td>0–15</td>
<td>14$\frac{1}{2}$</td>
<td>0.3683$p_x$</td>
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</tr>
<tr>
<td></td>
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</tbody>
</table>
A gear train is a set or system of gears arranged to transfer rotational torque from one part of a mechanical system to another.

Gear trains consists of:

- Driving gears - attached to the input shaft.
- Driven gears/Motor gears - attached to the output shaft.
- Idler gears - interposed between the driving and driven gear in order to maintain the direction of the output shaft the same as the input shaft or to increase the distance between the drive and driven gears.

A compound gear train refers to two or more gears used to transmit motion.
Gear train
The speed ratio between gear 5 and 1

\[
\frac{n_5}{n_1} = \left( -\frac{N_1}{N_2} \right) \left( -\frac{N_3}{N_4} \right) \left( -\frac{N_4}{N_5} \right)
\]

The minus signs indicate that the pinion and gear rotate in opposite directions.
Consider a pinion 2 driving a gear 3.

The speed of the driven gear is

\[ n_3 = \left| \frac{N_2}{N_3} \right| n_2 = \left| \frac{d_2}{d_3} \right| n_2 \]

\( d = \) pitch diameter

Gears 2, 3, and 5 are drivers.

Gears 3, 4, and 6 are driven members.

\[ n_6 = -\frac{N_2}{N_3} \frac{N_3}{N_4} \frac{N_5}{N_6} n_2 \]
Gear train

- The train value $e$ is

\[
e = \frac{\text{product of driving tooth numbers}}{\text{product of driven tooth numbers}}
\]

- For spur gears, $e$ is positive if the last gear rotates in the same sense as the first, and the negative if the last rotates in the opposite sense.

\[
n_L = en_F
\]

- $n_L$ = the speed of the last gear in the train, $n_F$ = the speed of the first.

- As a rough guideline, a train value of up to 10 to 1 can be obtained with one pair of gears. A two-stage compound gear train can obtain a train value of up to
Example 13-4

$$N_p = \frac{2(1)}{(1 + 2(4)\sin^2 20^\circ)} \left(4 + \sqrt{4^2 + (1 + 2(4)\sin^2 20^\circ)}\right)$$

$$N_p = 15.4 = 16\text{teeth}$$

A gearbox is needed to provide a 30:1 (± 1 percent) increase in speed, while minimizing the overall gearbox size. Specify appropriate teeth numbers.

Since the ratio is greater than 10:1, but less than 100:1, a two-stage compound gear train, such as in Figure 13–28, is needed. The portion to be accomplished in each stage is $\sqrt{30} = 5.4772$. For this ratio, assuming a typical $20^\circ$ pressure angle, the minimum number of teeth to avoid interference is 16, according to Eq. (13–11). The number of teeth necessary for the mating gears is $\frac{16\sqrt{30}}{16} = 87.64 \div 88$

From Eq. (13–30), the overall train value is

$$e = (88/16)(88/16) = 30.25$$

This is within the 1 percent tolerance. If a closer tolerance is desired, then increase the pinion size to the next integer and try again.
Example 13-4

A gearbox is needed to provide an exact 30:1 increase in speed, while minimizing the overall gearbox size. Specify appropriate teeth numbers.

The previous example demonstrated the difficulty with finding integer numbers of teeth to provide an exact ratio. In order to obtain integers, factor the overall ratio into two integer stages.

\[ e = 30 = (6)(5) \]

\[ \frac{N_2}{N_3} = 6 \quad \text{and} \quad \frac{N_4}{N_5} = 5 \]

With two equations and four unknown numbers of teeth, two free choices are available. Choose \( N_3 \) and \( N_5 \) to be as small as possible without interference. Assuming a 20° pressure angle, Eq. (13-11) gives the minimum as 16.

Then

\[ N_2 = 6 \quad N_3 = 6(16) = 96 \]

\[ N_4 = 5 \quad N_5 = 5(16) = 80 \]

The overall train value is then exact.

\[ e = (96/16)(80/16) = (6)(5) = 30 \]
A compound reverted gear train

- This requires the distances between the shafts to be the same for both stages on the train. The distance constraint is:

\[
\frac{d_2}{2} + \frac{d_3}{2} = \frac{d_4}{2} + \frac{d_5}{2}
\]

- The diametral pitch relates the diameters and the numbers of teeth,

\[
P = \frac{N}{d}
\]
A compound reverted gear train

- Replacing all the diameters gives

\[
\frac{N_2}{(2P)} + \frac{N_3}{(2P)} = \frac{N_4}{(2P)} + \frac{N_5}{(2P)}
\]

- Assuming a constant diametral pitch in both stages, and the geometry condition stated in terms of numbers of teeth:

\[
N_2 + N_3 = N_4 + N_5
\]
Planetary gear train
Planetary gear train

- Planetary trains always include a sun gear, a planet carrier or arm, and one or more planet gears.

- A planetary train composed of sun gear 2, an arm or carrier 3, and planet gears 4 and 5. The angular velocity of gear 2 relative to the arm in rev/min is:

\[ n_{23} = n_2 - n_3 \]

- The velocity of gear 5 relative to the arm is:

\[ n_{53} = n_5 - n_3 \]
Planetary gear train

- This equation expresses the ratio of gear 5 to that of gear 2, and both velocities are taken relative to the arm.

\[
\frac{n_{53}}{n_{23}} = \frac{n_5 - n_3}{n_2 - n_3}
\]

- The train value is

\[
e = \frac{n_5 - n_3}{n_2 - n_3}
\] or

\[
e = \frac{n_L - n_A}{n_F - n_A}
\]

- \(n_F\) = rev/min of first gear
- \(n_L\) = rev/min of last gear
Force analysis (Spur gearing)

- Designate the shafts using letters of the alphabet, a, b, c, etc.
- Figure shows a pinion mounted on shaft a rotating clockwise at $n_2$ rev/min and driving a gear on shaft b at $n_3$ rev/min.
Force analysis (Spur gearing)

- **Fa2 Force**: The force of gear 2 against a shaft a or the force of a shaft a against gear 2 is \( F_{a2} \).
- **F32 Force**: The force exerted by gear 2 against gear 3 as \( F_{23} \). \( F_{32} \) is the force exerted by gear 3 against the pinion.
- **Ta2 Torque**: The force exerted by gear 2 against gear 3 or the force of gear 2 against gear 3 is \( T_{a2} \).

**Notes**:
- \( r = \) the radial direction.
- \( t = \) the tangential direction.
The transmitted load

\[ F_{32}^r \text{ does not transmit power} \]

\[ W_t = F_{32}^t \]

\[ H = T\omega = \left( \frac{W_t d}{2} \right) \omega \]

\[ W_t = \frac{60000H}{\pi dn} \text{ kN} \]

\[ V = \pi dn \]

\[ T = T_{a2} = \frac{d}{2} W_t \]

\[ d = d_2 \]
Example 13-7

- Pinion 2 runs at 1750 rev/min and transmits 2.5 kW to idler gear 3. The teeth are cut on the 20° full-depth system and have a module of \( m = 2.5 \) mm. Draw a free body diagram of gear 3 and show all the forces that act upon it.

- Solution

The pitch diameter of gears 2 and 3 are

\[
d_2 = N_2m = 20(2.5) = 50\text{mm} \\
d_3 = N_3m = 50(2.5) = 125\text{mm}
\]

The transmitted load

\[
W_t = F_{23}^{t} = \frac{60000H}{\pi d_2 n} \\
= \frac{60000(2.5)}{\pi(50)(1750)} \\
= 0.546kN
\]
Example 13-7

\[ F_{43}^r = F_{23}^r = 0.199kN \]

\[ F_{43} = F_{23} = 0.581kN \]

\[ F^t = F^t = W_t = 0.546kN \]

\[ W_t = F^t = \frac{60000H}{\pi d_2 n} \]

\[ = \frac{60000(2.5)}{\pi(50)(1750)} = 0.546kN \]

\[ H = 2.5kW \]

\[ F_{23}^r = F_{23}^t \tan 20^\circ = (0.546) \tan 20^\circ \]

\[ = 0.199kN \]

\[ F_{23} = \frac{F_{23}^t}{\cos 20^\circ} \]

\[ = \frac{0.546}{\cos 20^\circ} = 0.581kN \]
**Example 13-7**

\[ F_{b3}^x = -(F_{23}^t + F_{43}^r) = -(-0.546 + 0.199) = 0.347 \text{kN} \]

\[ F_{b3}^y = -(F_{23}^r + F_{43}^t) = -(-0.199 - 0.546) = 0.347 \text{kN} \]

\[ F_{b3} = \sqrt{(0.347)^2 + (0.347)^2} = 0.491 \text{kN} \]
Force analysis-bevel gearing

\[ W_t = \frac{T}{r_{av}} \]

\[ W_a = W_t \tan \phi \sin \gamma \]

\[ W_r = W_t \tan \phi \cos \gamma \]
Force analysis-helical gearing

\[ W = \frac{W_t}{\cos \phi_n \cos \psi} \]

\[ W_r = W \sin \phi_n \]

\[ W_r = W_t \tan \phi_t \]

\[ W_t = \cos \phi_n \cos \psi \]

\[ W_a = W \cos \phi_n \sin \psi \]

\[ W_a = W_t \tan \psi \]
Force analysis-worm gearing

\[
W^z = W (\cos \phi_n \cos \lambda + f \sin \lambda)
\]

\[
W^z = W \cos \phi_n \cos \lambda
\]

\[
W^y = W \sin \phi_n
\]

\[
W^x = W \cos \phi_n \sin \lambda
\]

\[
W^x = W (\cos \phi_n \sin \lambda + f \cos \lambda)
\]
Velocity components in worm gearing

\[ V_s = \frac{V_W}{\cos \lambda} \]
SPUR GEAR BENDING
Based on ANSI/AGMA 2001-D04

\[ d_p = \frac{N_p}{P_d} \]

\[ V = \frac{\pi d n}{12} \]

\[ W' = \frac{33,000}{V} \]

Gear bending stress equation
Eq. (14–15)

\[ \sigma = W'K_KK_T \frac{P_d}{F} K_mK_R \]

1 [or Eq. (a), Sec. 14–10]; p. 759
Eq. (14–30); p. 759
Eq. (14–40); p. 764
Fig. 14–6; p. 753
Eq. (14–27); p. 756

Table below

\[ 0.99(S_1)_{107} \] Tables 14–3, 14–4; pp. 748, 749

Gear bending endurance strength equation
Eq. (14–17)

\[ \sigma_{all} = S_t \frac{Y_N}{K_TK_R} \]

1 if \( T < 250^\circ \text{F} \)

Fig. 14–14; p. 763

Bending factor of safety
Eq. (14–41)

\[ S_F = S_t \frac{Y_N}{(K_TK_R)} \]

Remember to compare \( S_F \) with \( S_H^2 \) when deciding whether bending or wear is the threat to function. For crowned gears compare \( S_F \) with \( S_H^3 \).

Table of Overload Factors, \( K_o \)

<table>
<thead>
<tr>
<th>Power source</th>
<th>Uniform</th>
<th>Moderate shock</th>
<th>Heavy shock</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform</td>
<td>1.00</td>
<td>1.25</td>
<td>1.75</td>
</tr>
<tr>
<td>Light shock</td>
<td>1.25</td>
<td>1.50</td>
<td>2.00</td>
</tr>
<tr>
<td>Medium shock</td>
<td>1.50</td>
<td>1.75</td>
<td>2.25</td>
</tr>
</tbody>
</table>

Driven Machine
SPUR GEAR WEAR
Based on ANSI/AGMA 2001-D04

\[ d_p = \frac{N_p}{T_d} \]

\[ V = \frac{\pi d n}{12} \]

\[ W' = 33000 \text{ H} \]

\[ \sigma_c = C_p \left( \frac{W' K_o K_m K_d}{d_p F} \right)^{1/2} \]

1 [or Eq. (a), Sec. 14–10]; p. 759
Eq. (14–30); p. 759
Eq. (14–23); p. 755
Eq. (14–27); p. 756

Gear contact stress equation

Eq. (14–13), Table 14–8; pp. 744, 757

Table below

0.99(\(S_c\))_{10}^7
Tables 14–6, 14–7; pp. 751, 752

Fig. 14–15; p. 763

Gear contact endurance strength

\[ \sigma_{c, all} = \frac{S_c Z_N C_H}{S_H K_T K_R} \]

Section 14–12, gear only; pp. 761, 762

Table 14–10, Eq. (14–38); pp. 763, 764
1 if \( T < 250^\circ \text{F} \)

Wear factor of safety

\[ S_H = \frac{S_c Z_N C_H}{(K_T K_R)} \]

Gear only

Remember to compare \( S_F \) with \( S_H^2 \) when deciding whether bending or wear is the threat to function. For crowned gears compare \( S_F \) with \( S_H^3 \).

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</tbody>
</table>
Spur and Helical gears

Objectives

- To analysis and design of spur and helical gears to resist bending failure of the teeth as well as pitting failure of tooth surface.

- Failure by bending will occur when the significant tooth stress equals or exceeds either the yield strength or the bending endurance strength.
The American Gear Manufacturers Association (AGMA) has for many years been the responsible authority for the dissemination of knowledge pertaining to the design analysis of gearing.
Wilfred Lewis introduced an equation for estimating the bending stress in gear teeth in which the tooth form entered into the formula.

The equation, announced in 1892, still remains the basis for most gear design today.
The Lewis Bending Equation

To derive the basic Lewis equation refer to Figure, which shows a cantilever of cross-sectional dimensions F and t, having a length l and a load W, uniformly distributed across the face width F.

- The section modulus: \( I/c = Ft^2/6 \)
- The bending stress \((\sigma)\).
The Lewis bending equations

\[
\sigma = \frac{M}{I/c} \Rightarrow I = c \left(\frac{Ft^3}{2}\right) = \frac{Ft^2}{6} = \frac{6W'l}{Ft^2}
\]

\[
\sigma = \frac{6W'l}{Ft^2} = \frac{W'}{F} \frac{1}{t^2 / 6l} = \frac{W'}{F} \frac{1}{t^2 / 4\delta} \Rightarrow 4x \frac{1}{2/3} = 4x \frac{3}{2} = 6
\]

\[
\frac{t/2}{x} = \frac{l}{t/2}
\]

\[
x = \frac{t^2}{4l}
\]
The Lewis bending equations

\[ \sigma = \frac{W' p}{F \left( \frac{2}{3} \right) \tau p} \Rightarrow y = \frac{2x}{3p} \]

\[ \sigma = \frac{W' p}{F p y} \]

\[ P = \frac{\pi}{p}, Y = \pi y \]

\[ \sigma = \frac{W' P}{F Y} \Rightarrow Y = \frac{2xP}{3} \]

Y means that only the bending of the tooth is considered and that the compression due to the radial component of the force is neglected.
# Values of the Lewis form factor \( Y \)

<table>
<thead>
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<th>( Y )</th>
<th>Number of Teeth</th>
<th>( Y )</th>
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<tr>
<td>26</td>
<td>0.346</td>
<td>Rack</td>
<td>0.485</td>
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*Values of the Lewis Form Factor \( Y \) (These Values Are for a Normal Pressure Angle of 20°, Full-Depth Teeth, and a Diametral Pitch of Unity in the Plane of Rotation)*
Dynamic effects

- When a pair of gears is driven at moderate or high speed and noise is generated, it is certain that dynamic effects are present.
- If a pair of gears failed at 500 lbf tangential load at zero velocity and at 250 lbf at velocity $V_1$, then a velocity factor, designated $K_v$, of 2 was specified for the gears at velocity $V_1$. 
**Dynamic effects**

- $K_v =$ the velocity factor
- $V =$ the pitch-line velocity in ft/min

\[
K_v = \frac{600 + V}{600} \quad (\text{cast iron, cast profile})
\]
\[
K_v = \frac{1200 + V}{1200} \quad (\text{cut or mild profile})
\]
\[
K_v = \frac{50 + \sqrt{V}}{50} \quad (\text{hobbed or shaped profile})
\]
\[
K_v = \frac{\sqrt{78 + \sqrt{V}}}{78} \quad (\text{shaved or ground profile})
\]

- SI units

\[
K_v = \frac{3.05 + V}{3.05} \quad (\text{cast iron, cast profile})
\]
\[
K_v = \frac{6.1 + V}{6.1} \quad (\text{cut or mild profile})
\]
\[
K_v = \frac{3.56 + \sqrt{V}}{3.56} \quad (\text{hobbed or shaped profile})
\]
\[
K_v = \frac{\sqrt{5.56 + \sqrt{V}}}{5.56} \quad (\text{shaved or ground profile})
\]

\[
\sigma = \frac{K_v W^t P}{FY}
\]
\[
\sigma = \frac{K_v W^t}{FmY}
\]
Example 14-1

A stock spur gear is available having a module of 3mm, a 38 mm face width, 16 teeth, and a pressure angle of 20° with full-depth teeth. The material is AISI 1020 steel in as-rolled condition. Use a design factor of $n_d = 3$ to rate the power output of the gear corresponding to a speed of 20 rev/s and moderate applications.
Example 14-1

Solution

- The term moderate applications seems to imply that the gear can be rated by using the yield strength as a criterion of failure. From Table A-18, $S_{ut} = 380 \text{ MPa}$ and $S_y = 210 \text{ MPa}$.

- A design factor of 3 means that the allowable bending stress is $210/3 = 70 \text{ MPa}$.

- The pitch diameter is $N_m = 16 (3) = 48 \text{ mm}$.

- The pitch-line velocity is $V = \pi d_n = \pi (0.049) 20 = 3.02 \text{ m/s}$.
Example 14-1

- The velocity factor (Eq 14-66):
  \[ K_v = \frac{6.1 + V}{6.1} = \frac{6.1 + 3.02}{6.1} = 1.5 \]

- Table 14-2, \( Y = 0.296 \) for 16 teeth.

- The tangential component of load \( W_t \)
  \[ W_t = \frac{mF \sigma_{all}}{K_v} = \frac{(0.003)(0.038)(0.296)(70) \times 10^6}{1.5} = 1574.72 \text{N} \]

- The power that can be transmitted is
  \[ H_p = W_t \times V = 1574.72(3.02) = 4755.6544 \text{W} \]
Surface durability

- Wear is the failure of the surfaces of gear teeth.
- Pitting is a surface fatigue failure due to many repetitions of high contact stresses.
- Scoring is a lubrication failure, and abrasion, which is wear due to the presence of foreign material.
Surface durability

To obtain an expression for the surface-contact stress, we shall employ the Hertz theory. The contact stress between two cylinders may computed from the equation;

\[ p_{\text{max}} = \frac{2F}{\pi bl} \]

- \( p_{\text{max}} \) = largest surface pressure.
- \( F \) = force pressing the two cylinders together.
- \( L \) = length of cylinders.
Surface durability

- Half-width \( b \) is obtained from

\[
b = \left\{ \frac{2F}{\pi l} \left[ \frac{(1 - v_1^2)/E_1}{(1/d_1)} + \frac{(1 - v_1^2)/E_2}{(1/d_2)} \right] \right\}^{1/2}
\]

- The surface compressive stress (Hertzian stress)

\[
\sigma_C^2 = \left\{ \frac{W^t}{\pi F \cos\phi} \left[ \frac{(1/r_1) + (1/r_2)}{[(1 - v_1^2)/E_1] + [(1 - v_1^2)/E_2]} \right] \right\}^{1/2}
\]
Surface durability

- The radii of curvature of the tooth profiles at the pitch point are

\[ r_1 = \frac{d_p \sin \phi}{2}, \quad r_2 = \frac{d_G \sin \phi}{2} \]

- An elastic coefficient \( C_p \)

\[
C_p = \left[ \frac{1}{\pi \left( \frac{1 - v_p^2}{E_p} + \frac{1 - v_G^2}{E_G} \right)} \right]^{1/2}
\]

\[
\sigma_c = -C_p \left[ \frac{K_v W^t}{F \cos \phi} \left( \frac{1}{r_1} + \frac{1}{r_2} \right) \right]^{1/2}
\]
Two fundamental stress equations are used in the AGMA methodology, one for bending stress and another for pitting resistance (contact stress).

In AGMSA terminology, these are called stress numbers, as contrasted with actual applied stress ($\sigma$).
AGMA strength equations

\[ \sigma = W^t K_0 K_v K_s \frac{P_d}{F} \frac{K_m K_B}{J} \]  (US Unit)

\[ \sigma = W^t K_0 K_v K_s \frac{1}{b m_t} \frac{K_H K_B}{Y_t} \]  (SI units)

- Pitting resistance

\[ \sigma_C = C_p \sqrt{W^t K_0 K_v K_s \frac{K_m}{d_p F} \frac{C_f}{J}} \]  (US Unit)

\[ \sigma_C = Z_E \sqrt{W^t K_0 K_v K_s \frac{K_H}{d_{w1} b} \frac{Z_R}{Z_I}} \]  (SI Unit)
AGMA strength equations

- The allowable bending stress

\[
\sigma = \frac{S_t \ Y_N}{S_F \ K_T K_R} \quad (US\ Unit)
\]

\[
\sigma = \frac{S_t \ Y_N}{S_F \ Y_\theta Y_Z} \quad (SI\ Unit)
\]

- The allowable contact stress \( \sigma_{C, all} \)

\[
\sigma_{C, all} = \frac{S_C \ Z_N C_H}{S_H K_T K_R} \quad (US\ Unit)
\]

\[
\sigma_{C, all} = \frac{S_C \ Z_N Z_W}{S_H Y_\theta Y_Z} \quad (SI\ Unit)
\]
Bevel gears

Bevel gears may be classified:

- Spiral bevel gears
- Zerol bevel gears
- Hypoid gears
- Spiroid gears
Straight bevel gears
The pitch angles are defined by the pitch cones meeting at the apex. They are related to the tooth numbers as follows:

\[ \tan \gamma = \frac{N_P}{N_G} \]
\[ \tan \Gamma = \frac{N_G}{N_P} \]

The virtual number of teeth \((N')\) and \(p\) is the circular pitch.

\[ N' = \frac{2\pi r_b}{p} \]
Spiral bevel gears
Zerol bevel gears
Hypoid bevel gears
Bevel-gear stresses and strengths

- Fundamental contact stress equation

\[
S_c = \sigma_c = C_p = \left( \frac{W^t}{Fd_p I} K_o K_v K_m C_s C_{xc} \right)^{\frac{1}{2}} \text{ (US units)}
\]

\[
\sigma_H = Z_E \left( \frac{1000W^t}{bdZ_1} K_A K_v K_{H\beta} Z_x Z_{xc} \right)^{\frac{1}{2}} \text{ (SI units)}
\]
Bevel-gear stresses and strengths

- Permissible contact stress number (strength) equation.
- Bending Stress.
- Permissible bending stress equation.
AGMA equation factors

- Overload factor $K_o (K_A)$
- Safety factors $S_H$ and $S_F$
- Dynamic factor $K_v$
- Size factor for pitting resistance $C_s (Z_x)$
- Size factor for bending $K_s (Y_x)$
- Dynamic factor $K_v$
- Size factor for pitting resistance $C_s (Z_x)$
- Size factor for bending $K_s (Y_x)$
- Load distribution factor $K_m(K_{H\beta})$
AGMA equation factors

- Crowning factor for pitting $C_{xc} (Z_{xc})$.
- Lengthwise curvature factor for bending strength $K_x (Y_\beta)$.
- Pitting resistance geometry factor I ($Z_1$).
- Bending strength geometry factor J ($Y_j$).
- Stress-cycle factor for pitting resistance $C_L (Z_{NT})$.
- Stress-cycle factor for bending strength $K_L (Y_{NT})$. 
AGMA equation factors

- Hardness-ratio factor $C_H (Z_W)$.
- Temperature factor $K_T (K_\theta)$.
- Reliability factors $C_R (Z_Z)$ and $K_R (Y_Z)$.
- Elastic coefficient for pitting resistance $C_p (Z_E)$.
- Allowable contact stress.
- Allowable bending stress numbers
- Reversed Loading.
Example 15-1

A pair of identical straight-tooth miter gears listed in a catalog has a module of 5 at the large end, 25 teeth, a 27.5 mm face width, and a 20° normal pressure angle; the gear are grade 1 steel through-hardened with a core and case hardness of 180 Brinell. The gear are uncrowned and intended for general industrial use. The have quality number of $Q_v = 7$. It is likely that the application intended will require outboard mounting of the gears. Use a safety factor of 1, a $10^7$ cycle life, and a 0.99 reliability.
Straight bevel gear analysis

a) For a speed of 600 rev/min, find the power rating of this gearset based on AGMA bending strength.
   \[ H = W^t V_{et} \]

b) For the same conditions as in part (a) find the power rating of this gearset based on AGMA wear strength.
   \[ H = W^t V_{et} \text{(Wear)} \]

c) For reliability of 0.995 a gear life of $10^9$ revolutions, and a safety factor of $S_F = S_H = 1.5$, find the power rating for this gearset using AGMA strengths.
   \[ H = W^t V_{et} \text{(life goal } 10^9 \text{ cycles)} \]
Straight bevel gear analysis

Solution

Figure 15-14 and 15-15 show roadmap summary of principal straight bevel gear bending equations and their parameters.

- \(d_p = n_p \ met = 25(5) = 125 \text{ mm}\)
- \(V_{et} = \pi d_p n_p / 60 = \pi (0.125)(600)/60 = 3.93 \text{ m/s}\)
- Overload factor \(\Rightarrow K_A = 1.00\) (Table 15-2), uniform-uniform loading.
- Safety factor; \(S_F = 1\), \(S_H = 1\)
- Dynamic factor \(K_v\) from Eq. 15.6
Straight bevel gear analysis

- \( B = 0.25 \times (12 - 7)^{2/7} = 0.731 \)
- \( A = 50 + 56(1 - 0.731) = 65.06 \)

\[
K_v = \left( \frac{A + \sqrt{200V_{et}}}{A} \right)^B = \left( \frac{65.06 + \sqrt{200(3.93)}}{65.06} \right)^{0.731} = 1.299
\]

- \( V_{et \ max} = \left[ A + (Qv - 3) \right]^2 / 200 \)
  
  \[
  = \left[ 65.06 + (7 - 3) \right]^2 / 200 = 23.8 \text{ m/s}
  \]

- \( V_{et} < V_{et \ max}, \ 3.93 < 23.8 \text{ m/s} \)
- \( K_v \) is valid (Eq 15-10)

\[
Y_x = 0.487 + 0.008 \ 339 \ m_{et}, \ 1.6 \leq m_{et} \leq 50 \ \text{mm}
\]

\[
Y_x = 0.487 + 0.008 \ 339 \ (5) = 0.528
\]
Straight bevel gear analysis

Eq 15-11, \( K_{mb} = 1.25 \) and \( K_{HB} = 1.25 + 5.6(10^{-6})(27.5)^2 = 1.254 \)

Eq 15-13, \( Y^B = 1 \). Eq 15-6, \( Z_1 = 0.065 \); from 15-7, \( Y_j = 0.216 \), \( J_G = 0.216 \).

Eq 15-15, \( Y_{NT} = 1.683 \times 10^7 \times 0.0323 = 0.99996 = 1 \)

From Eq (15-14)
\[ Z_{NT} = 3.4822(10^7)^{-0.0602} = 1.32 \]

Since \( H_{B1}/H_{B2} = 1 \), then from Fig. 15-10, \( Z_w = 1 \). From Eq. 15-13 and 15-18, \( Y_B = 1 \) and \( K_0 = 1 \), respectively.

Eq 15-20, \( Y_Z = 0.70 - 0.15\log(1-0.99) = 1 \), \( Z_z = \sqrt{Y_Z} = \sqrt{1} = 1 \)
Straight bevel gear analysis

a) Bending: From Eq. 15-23

\[ \sigma_{Film} = 0.3 \times (180) + 14.48 = 68.48 \text{ Mpa} \]

Eq 15-3

\[
\sigma_F = \frac{1000W^t}{b} \frac{K_A K_V Y_X K_{HB}}{m_{et} Y_B Y_j} = \frac{W^t}{27.5(5)(1)(1.299)} \frac{0.528(1.254)}{(1)(0.216)} = 0.029W^t
\]

\[
\sigma_{FP} = \frac{\sigma_{Flim} Y_{NT}}{S_F K_\theta Y_Z} = \frac{64.48(1)}{(1)(1)(1)} = 64.48\text{MPa}
\]
Straight bevel gear analysis

0.029 $W_t = 64.48$

$W_t = 2223$ N

$H = W_t \cdot V_{et} = 2223 \cdot (3.93) = 8736$ W

b) Wear

From Fig 15-12, $\sigma_{H\text{lim}} = 2.35(180) + (162.89) = 585.9$ Mpa

From Eq. 15-12

$$\sigma_{HP} = \frac{(\sigma_{H\text{lim}})_P Z_{NT} Z_W}{S_H K_\theta Z_Z} = \frac{585.9(1.32)(1)}{(1)(1)(1)} = 773.4 \text{MPa}$$
Straight bevel gear analysis

\[ Z_E = 190 \sqrt{N/mm^2} \]
\[ Z_X = 0.004(92)(27.5) + (0.4375) = 0.573 \]

From Eq (15-12), \( Z_{XC} = 2 \), substituting in Eq (15-1):

\[ \sigma_H = Z_E \left( \frac{1000 W^t}{bdZ_1} K_A K_v K_H \beta Z_x Z_{xc} \right)^{1/2} \text{(SI units)} \]

\[ \sigma_H = (190) \left( \frac{W^t}{(27.5)(125)(0.065)} (1)(1.299)(1.254)(0.573)(2) \right)^{1/2} \]
Equating $\sigma_H$ and $\sigma_{HP}$ gives

$$17.37\sqrt{W^t} = 773.4$$

$W^t = 1982$

$H = W^t \ V_{et} = 1982 (3.93) = 7789$ W

Rated power for the gearset is

$H = \text{min} \ (8736, 7789) = 7789$ Watt

C) **Life goal $10^9$ cycles**, $R = 0.995$, $S_F = S_H = 1.5$ and from Eq 15-15

$$Y_{NT} = 1.683 \ (10^9)^{-0.0323} = 0.8618$$
Straight bevel gear analysis

From Eq 15-19
\[ Y_Z = 0.50 - 0.25 \log(1 - 0.995) = 1.075 \]
\[ Z_Z = \sqrt{Y_Z} = \sqrt{1.075} = 1.037 \]

From Eq 15-14
\[ Z_{NT} = 3.482(10^9)^{-0.0602} = 1 \]

**Bending;**

From Eq 15-23 and part (a), \( \sigma_{F_{lim}} = 64.48 \) MPa.

From 15-3
\[
\sigma_F = \frac{1000W^t}{27.5(5)} (1)(1.299) \frac{(0.528)(1.254)}{(1)(0.216)} = 0.029 W^t
\]
Straight bevel gear analysis

From Eq 15-14

\[
\sigma_{FP} = \frac{\sigma_{F \text{lim}} Y_{NT}}{S_F K_\theta Y_Z} = \frac{64.48 \times (0.8616)}{(1.5)(1)(1.075)} = 34.5 \text{MPa}
\]

Equating \(\sigma_F\) to \(\sigma_{FP}\)

0.029 \(W^t\) = 34.5 \(\Rightarrow\) \(W^t\) = 1190 N

\(H = 1190 \times (3.73) = 4438.7\) Watt

Wear

From Eq 15-22 and part (b), \(\sigma_{H \text{lim}} = 585.9\) MPa
Straight bevel gear analysis

From Eq 15-2

\[ \sigma_{HP} = \frac{(\sigma_{Hlim})_P Z_{NT} Z_W}{S_H K_\theta Z_Z} = \frac{585.9(1)(1)}{(1.5)(1)(1.037)} = 376.7 \text{MPa} \]

Substituting into Eq 15-1 gives part (b), \( \sigma_{Hlim} = 17.37 \sqrt{W^t} \)

Equating \( \sigma_H \) to \( (\sigma_H)_P \)

\[ 376.7 = 17.37 \sqrt{W^t} \Rightarrow W^t = 470 \text{ N} \]

The wear power is

\[ H = 470 (3.73) = 1753 \text{ W} \]

\[ H = \min (4438.7, 1753) = 1753 \text{ Watt} \]
Design of a straight bevel gear mesh

- A useful decision set for straight-bevel gear design is

A priori decisions

- Function
- Design factor
- Tooth system
- Tooth count

Design variables

- Pitch and face width
- Quality number
- Gear material, core and case hardness
- Pinion material, core and case hardness.
Worm gear analysis

Example 15-3

A single-thread steel worm rotates at 1800 rev/min, meshing with a 24-tooth worm gear transmitting 3 hp to the output shaft. The worm pitch diameter is 3 in and the tangential diametral pitch of the gear is 4 teeth/in. The normal pressure angle is 14.5°. The ambient temperature is 70°F. The application factor is 1.25 and the design factor is 1; gear face width is 2 in, lateral case area 600 in², and the gear is chill-cast bronze.
Worm gear analysis

a) Find the geometry
b) Find the transmitted gear forces and the mesh efficiency.
c) Is the mesh sufficient to handle the loading?
d) Estimate the lubricant sump temperature.

Solution

a) Find the geometry

\[ m_G = \frac{N_G}{N_W} = \frac{24}{1} = 24 \]

Gear: \[ D = \frac{N_G}{P_t} = \frac{24}{4} = 6.000 \text{ in} \]

Worm: \[ d = 3.000 \text{ in} \]
Worm gear analysis

The axial circular pitch $p_x$ is $p_x = \frac{\pi}{p_t} = \frac{\pi}{4} = 0.7854$ in

$C = \frac{3+6}{2} = 4.5$ in

Eq 15-39, $a = \frac{p_x}{\pi} = 0.7854/\pi = 0.250$ in

Eq 15-40, $b = 0.3683 \times p_x = 0.3683 (0.7854) = 0.289$ in

Eq 15-41, $h_t = 0.6866 \times p_x = 0.6866 (0.7854) = 0.539$ in

Eq 15-42, $d_o = 3 + 2(0.250) = 3.500$ in

Eq 15-43, $d_r = 3 - 2(0.289) = 2.422$ in

Eq 15-44, $D_t = 6 + 2(0.250) = 6.500$ in

Eq 15-45, $D_r = 6 - 2(0.289) = 5.422$ in

Eq 15-46, $c = 0.289 - 0.250 = 0.039$ in
Eq 15-47, \((F_W)_{\text{max}} = 2\sqrt{2}(6)(0.250) = 3.464\) in

The tangential speeds of the worm, \(V_W\), and gear, \(V_G\), are respectively.

\(V_W = \pi(3)\frac{1800}{12} = 1414\) ft/min.

\(V_G = \frac{\pi(6)(1800)}{24} = 117.8\) ft/min

The lead of the worm, from Eq 13-27, is \(L = p_x N_W = 0.7854(1) = 0.7854\) in.

The lead angle \(\lambda\) from Eq 13-28 is

\[\lambda = \tan^{-1} \frac{L}{\pi d} = \tan^{-1} \frac{0.7854}{\pi(3)} = 4.764^\circ\]

The normal diametral pitch for a gear is the same as for a helical gear, which from Eq 13-18 with \(\Psi = \lambda\) is…
Worm gear analysis

\[ P_n = \frac{P_t}{\cos \lambda} = \frac{4}{\cos 4.764^\circ} = 4.014 \]

\[ p_n = \frac{\pi}{P_n} = \frac{\pi}{4.014} = 0.7827 \]

The sliding velocity, from Eq 15-62 is

\[ V_s = \frac{\pi d n_w}{12 \cos \lambda} = \frac{\pi (3) 1800}{12 \cos 4.764^\circ} = 1419 \text{ ft/min} \]
b) The coefficient of friction, from Eq 15-38 is
\[ F = 0.103 \exp[-0.110(1419)^{0.450}] + 0.012 = 0.0178 \]
The efficiency \( e \), from Eq 13-46, is
\[
\frac{\cos \phi_n - f \tan \lambda}{\cos \phi_n + f \cot \lambda} = \frac{\cos 14.5^\circ - 0.0178 \tan 4.764^\circ}{\cos 14.5^\circ + 0.0178 \cot 4.764} = 0.818
\]
The designer used \( n_d = 1 \), \( K_a = 1.25 \) and an output horsepower of \( H_o = 3 \) hp. The gear tangential force component \( W^t_G \), from Eq 15-58:
\[
W^t_G = \frac{33000 n_d H_o K_a}{V_G e} = \frac{33000(1)3(1.25)}{117.8(0.818)} = 1284 \text{ lbf}
\]
Worm gear analysis

The tangential force on the worm is given by Eq 15-57

\[ W_t = W_G \frac{\cos \phi_n \sin \lambda + f \cos \lambda}{\cos \phi_n \cos \lambda - f \sin \lambda} \]

\[ W_t = 1284 \frac{\cos 14.5^\circ \sin 4.764^\circ + 0.0178 \cos 4.764^\circ}{\cos 14.5^\circ \cos 4.764^\circ - 0.0178 \sin 4.764^\circ} = 131 \text{ lbf} \]
THANK YOU