1) LOAD AND STRESS ANALYSIS
   i. Principle stress
   ii. The maximum shear stress
   iii. The endurance strength of shaft.

1) Problem 3- 71
A countershaft carrying two-V belt pulleys is shown in the figure. Pulley A receives power from a motor through a belt with the belt tensions shown. The power is transmitted through the shaft and delivered to the belt on pulley B. Assume the belt tension on the loose side at B is 15 percent of the tension on the tight side.

   a) Determine the tensions in the belt on pulley B, assuming the shaft is running at a constant speed.
   b) Find the magnitudes of the bearing reaction forces, assuming the bearings act as simple supports.
   c) Draw shear-force and bending moment diagrams for the shaft. If needed, make one set for the horizontal plane and another set for the vertical plane.
   d) At the point of maximum bending moment, determine the bending stress and the torsional shear stress.
   e) At the point of maximum bending moment, determine the principal stresses and the maximum shear stress.

**Solution**
Assume the belt tension on the loose side at B is 15 percent of the tension on the tight side. \( T_2 = 0.15 T_1 \)
Problem 3–71*

Dimensions in millimeters.

a) Determine the tensions in the belt on pulley B, assuming the shaft is running at a constant speed.

\[ \sum T = 0 \]

\[ (300 - 45)N(125)mm + (T_2 - T_1)N(150)mm = 0 \]

\[ 31875Nmm + (0.15T_1 - T_1)N(150)mm = 0 \]

\[ 31875Nmm - 127.5T_1 = 0 \]
\[ T_1 = 250 Nmm \text{  On pulley B} \]
\[ T_2 = (0.15)250 Nmm = 37.5 Nmm \]
\[ T = T_1 + T_2 = 250 + 37.5 = 287.5 Nmm \]

b) Find the magnitudes of the bearing reaction forces, assuming the bearings act as simple supports.

**Y**  
31.82 N  
45 N  
31.82N  
\[ T_2 = 0.15 T_1 \]

**X**  
212.132 N  
300 N  
212.132 N

\[ \sum M_{Oy} = 0 \]
\[ 345 \cos 45^\circ (300) - 287.5 (700) + R_{Cz}(850) = 0 \]
\[ R_{Cz} = -150.7 N \]

\[ \sum F_z = 0 \]
\[ R_{Oz} - 345 \cos 45^\circ + 287.5 - 150.7 = 0 \]
\[ R_{Oz} = 107.2 N \]

\[ \sum M_{oz} = 0 \]
\[ 345 \sin 45^\circ (300) + R_{Cy}(850) = 0 \]
\[ R_{Cy} = -86.10 N \]
\[
\sum F_y = 0 \\
R_{Oy} + 345 \cos 45^\circ - 86.10 = 0 \\
R_{Oz} = -157.9 \text{ N}
\]
c) Draw shear-force and bending moment diagrams for the shaft. If needed, make one set for the horizontal plane and another set for the vertical plane.
d) At the point of maximum bending moment, determine the bending stress and the torsional shear stress.

The critical location is at A where both planes have the maximum bending moment. Combining the bending moments from the two planes;

\[ M = \sqrt{(-47.37)^2 + (-32.16)^2} = 57.26 \text{ Nm} \]

The torque transmitted through the shaft from A to B is

\[ T = (300 - 45)(0.125) = 31.88 \text{ Nm} \]

The bending stress and the torsional stress are both maximum are on the outer surface of a stress element.

\[ \sigma = \frac{Mc}{I} = \frac{32M}{\pi d^3} = \frac{32(57.26)}{\pi(0.020)^3} = 72.9 \times 10^6 \text{ Pa} = 72.9 \text{ MPa} \]

\[ \tau = \frac{Tr}{J} = \frac{16T}{\pi d^3} = \frac{16(31.88)}{\pi(0.020)^3} = 20.3 \times 10^6 \text{ Pa} = 20.3 \text{ MPa} \]
e) At the point of maximum bending moment, determine the principal stresses and the maximum shear stress.

\[
\sigma_1, \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau_{xy})^2} = \frac{72.9}{2} \pm \sqrt{\left(\frac{72.9}{2}\right)^2 + (20.3)^2} =
\]

\[
\sigma_1 = 78.2\text{MPa}
\]
\[
\sigma_2 = -5.27\text{MPa}
\]

\[
\tau_{max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau_{xy})^2} = \sqrt{\left(\frac{72.9}{2}\right)^2 + (20.3)^2} = 41.7\text{MPa}
\]
2) **Problem 3-73**

A gear reduction unit uses the countershaft shown in the figure. Gear A receives power from another gear with the transmitted force $F_A$ applied at the 20° pressure angle as shown. The power is transmitted through the shaft and delivered through gear B through a transmitted force $F_B$ at the pressure angle shown.

a) Determine the force $F_B$, assuming the shaft is running at a constant speed.

b) Find the magnitudes of the bearing reaction forces, assuming the bearings act as simple supports.

c) Draw shear-force and bending-moment diagrams for the shaft. If needed, make one set for the horizontal plane and another set for the vertical plane.

d) At the point of maximum bending moment, determine the bending stress and the torsional shear stress.

e) At the point of maximum bending moment, determine the principal stresses and the maximum shear stress.

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**Problem 3–73**
a) Determine the force $F_B$, assuming the shaft is running at a constant speed.

\[ \sum T = 0 \]
\[ -11000(\cos20^\circ)(300) - F_B(\cos25^\circ)(150) = 0 \]
\[ F_B = 22810 \text{ N} \]

b) Find the magnitudes of the bearing reaction forces, assuming the bearings act as simple supports.

\[ \sum M_{oz} = 0 \]
\[ -11000(\sin20^\circ)(400) - 22810(\sin25^\circ)(750) + R_{Cy}(1050) = 0 \]
\[ R_{Cy} = 8319 \text{ N} \]

\[ \sum F_y = 0 \]
\[ R_{Oy} - 11000(\sin20^\circ) - 22810(\sin25^\circ) + 8319 = 0 \]
\[ R_{Oy} = 5083 \text{ N} \]
\[ \sum M_{oy} = 0 \]
\[ 11000 (\cos 20^\circ)(400) - 22810 (\cos 25^\circ)(750) - R_{cz}(1050) = 0 \]
\[ R_{cz} = -10830 \, N \]

\[ \sum F_z = 0 \]
\[ R_{oz} - 11000 (\cos 20^\circ) + 22810 (\cos 25^\circ) - 10830 = 0 \]
\[ R_{oz} = 494 \, N \]
c) Draw shear-force and bending-moment diagrams for the shaft. If needed, make one set for the horizontal plane and another set for the vertical plane.

d)

\[ V (N) \]

\[ M (Nm) \]
d) At the point of maximum bending moment, determine the bending stress and the torsional shear stress.

The critical location is at B where both planes have the maximum bending moment. Combining the bending moments from the two planes:

\[ M = \sqrt{(2496)^2 + (3249)^2} = 4097 \text{ Nm} \]

The torque transmitted through the shaft from A to B is

\[ T = 11000(\cos20^\circ)(0.3) = 3101 \text{ Nm} \]

The bending stress and the torsional stress are both maximum are on the outer surface of a stress element.
\[
\sigma = \frac{Mc}{I} = \frac{32M}{\pi d^3} = \frac{32(4097)}{\pi(0.050)^3} = 333.9 \times 10^6 Pa = 333.9 MPa
\]
\[
\tau = \frac{Tr}{J} = \frac{16T}{\pi d^3} = \frac{16(3101)}{\pi(0.050)^3} = 126 \times 10^6 Pa = 20.3 MPa
\]

e) At the point of maximum bending moment, determine the principal stresses and the maximum shear stress.

\[
\sigma_1, \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau_{xy})^2} = \frac{333.9}{2} \pm \sqrt{\left(\frac{333.9}{2}\right)^2 + (126.3)^2}
\]

\[
\sigma_1 = 376 MPa
\]
\[
\sigma_2 = -42.4 MPa
\]

\[
\tau_{max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau_{xy})^2} = \sqrt{\left(\frac{333.9}{2}\right)^2 + (126.3)^2} = 209 MPa
\]

f) The endurance strength of the shaft:

Material: AISI 1040 CD with \( S_{ut} = 590 \) MPa

Diameter of shaft: \( d = 50 \) mm

Machined or cold-drawn \( a = 4.51 \) and \( b = -0.265 \)

\[
S_e' = 0.5(590) = 295 MPa
\]

\[
k_a = aS_{ut}^b = (4.51)(590)^{-0.265} = 0.83157
\]

Surface factor \( k_a = 0.83157 \)

The loading situation is rotating bending.

\[
k_b = \left(\frac{d}{7.62}\right)^{-0.107} = \left(\frac{50}{7.62}\right)^{-0.107} = 0.8176
\]

\[
S_e = k_ak_bS_e' = (0.83157)(0.8176)(295) = 200.568 MPa.
\]

The endurance strength is \( S_e = 200.568 MPa \)
3) Problem 11-34

A gear-reduction unit uses the countershaft depicted in figure. Find the two bearing reactions. The bearings are to be angular-contact ball bearings, having a desired life of 40 kh when used at 200 rev/min. Use 1.2 for the application factor and a reliability goal for the bearing pair of 0.95. Select the bearings from Table 11-2

Solution
b) Determine the force $F_B$, assuming the shaft is running at a constant speed.

$$\sum T = 0$$
$$-1068(\cos20°)(300) - F_B(\cos25°)(150) = 0$$
$$F_B = 2214.685 \text{ N}$$

c) Find the magnitudes of the bearing reaction forces, assuming the bearings act as simple supports.

$$\sum M_{oz} = 0$$
$$-1068(\sin20°)(400) - 2214.685(\sin25°)(750) + R_{Cy}(1050) = 0$$
$$R_{Cy} = 807.700 \text{ N}$$

$$\sum F_y = 0$$
$$R_{Oy} - 1068(\sin20°) - 2214.685(\sin25°) + 807.700 = 0$$
$$R_{Oy} = 493.543 \text{ N}$$
\[ \sum M_{oy} = 0 \]
\[ 1068(\cos20^\circ)(400) - 2214.685(\cos25^\circ)(750) - R_{Cz}(1050) = 0 \]
\[ R_{Cz} = -1051.38 \, N \]

\[ \sum F_z = 0 \]
\[ R_{Oz} - 1068(\cos20^\circ) + 2214.685(\cos25^\circ) - 1051.38 = 0 \]
\[ R_{Oz} = 48.405 \, N \]
\[ R_o = \sqrt{(R_{oy})^2 + (R_{oz})^2} = \sqrt{(493.543)^2 + (48.405)^2} \]
\[ R_o = \sqrt{245927.736} = 495.911 \, N \]

\[ R_c = \sqrt{(R_{cy})^2 + (R_{cz})^2} = \sqrt{(807.7)^2 + (1051.38)^2} \]
\[ R_o = \sqrt{1757779.19} = 1325.813 \, N \]
\[ F_D O = (1.2)(495.911) = 595.093 \, N \]
\[ F_D C = (1.2)(1325.813) = 1590.97 \, N \]
\[ x_D = \frac{L_D n_D 60}{L_R n_R 60} = \frac{(40000)(200)(60)}{(10^6)} = 480 \]

Realibility for \( R_A \) and \( R_B \) are \( \sqrt{R} = \sqrt{0.95} = 0.975 \)

\[
(C_{10})_O = 595.093 \left[ \frac{480}{0.02 + 4.439 \left[ \ln \left( \frac{1}{0.975} \right) \right]^{1.483}} \right]^{\frac{1}{3}} = 6351.465 \, N
\]

**Bearing Type is angular contact bearing at O, series number 02- 12**

Bore : 12 mm  
OD : 32 mm  
Width : 10 mm  
Fillet radius : 0.6 mm  
Shoulder diameter: \( d_s = 14.5 \) mm, \( d_H = 28 \) mm

\[
(C_{10})_O = 1590.97 \left[ \frac{480}{0.02 + 4.439 \left[ \ln \left( \frac{1}{0.975} \right) \right]^{1.483}} \right]^{\frac{1}{3}} = 16980.522 \, N
\]

**Bearing Type is angular contact bearing at C, series number 02- 30**

Bore : 30 mm  
OD : 62 mm  
Width : 16 mm  
Fillet radius : 1.0 mm  
Shoulder diameter: \( d_s = 35 \) mm, \( d_H = 55 \) mm