CO3: Ability to analyze balancing of machinery
A detail cross section of an Internal combustion engine.
The basic mechanism consist of a
- crank
- Connecting rod (coupler)
- piston (slider)

Multicylinder a vee-engine
FIGURE 14-1
Various multicylinder engine configurations
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FIGURE 14-2
Cutaway views of a four-stroke, four-cylinder inline engine  Courtesy of FIAT Corporation, Italy
FIGURE 14-3
Crankshaft from an inline four-cylinder engine with pistons, connecting rods, and flywheel
Illustration copyright Eaglemoss Publications/Car Care Magazine. Reprinted with permission.
FIGURE 14-4
Cross section of a BMW 5-liter V-12 engine and its power and torque curves  Courtesy of BMW of North America Inc.
FIGURE 14-5
Cutaway view of a 5-liter BMW V-12 engine  Courtesy of BMW of North America Inc.
FIGURE 14-6
Chevrolet Corvair horizontally opposed six-cylinder engine
_Courtesy of Chevrolet Division, General Motors Corp._
FIGURE 14-7
The Gnome rotary engine (circa 1915). Note the multiple connecting rods on the single stationary crank pin.\(^1\)
The crank phase diagram

\[ \Delta \phi_{\text{inertia}} = \frac{360^\circ}{n} \quad n \text{ is number of cylinders} \]

Four cylinder 0°, 90°, 180°, 270° in line engine

(a) Crankshaft phase angles
Four cylinder 0°, 90°, 180°, 270°

**FIGURE 14-8**
Crank phase angles and the phase diagram

**FIGURE 14-9**
The schematic crank phase diagram
Even Firing

In general, it is desirable to create a firing pattern among the cylinders that is evenly spaced in time. If the cylinders fire unevenly, vibrations will be created which may be unacceptable.

The optimum delta phase angle

\[
\Delta \phi_{\text{two stroke}} = \frac{360^\circ}{n}
\]

If the engine is a two-stroke, there will be one power pulses per Revolution in each of its \( n \) cylinders.

The optimum delta phase angle

\[
\Delta \phi_{\text{four stroke}} = \frac{720^\circ}{n}
\]

If the engine is a four-stroke, there will be one power pulses in each cylinders every two revolutions.
The delta power stroke angle for the two-stroke engine

\[ \Delta \psi_{two \ stroke} = \frac{360^\circ}{n} \]

The delta power stroke angle for the four-stroke engine

\[ \Delta \psi_{four \ stroke} = \frac{720^\circ}{n} \]
(a) Intake stroke  (b) Compression stroke  (c) Power stroke  (d) Exhaust stroke

(e) The gas pressure curve
To determine the firing pattern of an engine we must return to the crank phase. The power pulses for a two-stroke cycle, four-cylinder engine with the $\phi_i = 0, 90, 180, 270^\circ$ phase angle crank configuration.
The power pulses for a four-stroke cycle, four-cylinder engine with the \( \phi_i = 0, 90, 180, 270° \) phase angle crank configuration.
Phase angle Cyl. \\
0 1 \\
-180 2 \\
0 3 \\
-180 4 \\

the \( \phi_i = 0, 180, 0, 180^\circ \) phase angle crank configuration.

**FIGURE 14-16**

Even firing four-stroke, four-cylinder engine crank phase diagram with \( \phi_i = 0, 180, 0, 180^\circ \)
the $\phi_i = 0, 180, 180, 0^\circ$ phase angle crank configuration.

FIGURE 14-18
Even firing four-stroke, four-cylinder engine crank phase diagram with a mirror-symmetric $0, 180, 180, 0^\circ$ crankshaft
FIGURE 14-24
Four-stroke vee-eight crank phase diagram with 0, 90, 270, 180° crankshaft phase angles
Shaking forces in inline engines

The same design principles which apply to inline engines also apply to vee and opposed configurations.

A vee six is essentially two three-cylinder inline engines on a common crankshaft.

A vee eight is two four-cylinder inline.

The same criteria of even firing and inertia balance apply to its selection.
The shaking force

The shaking force for one cylinder with the crankshaft rotating at constant $\omega$.

$\alpha = 0$

$$F_s = \left[ m_A r \omega^2 \cos \omega t + m_B r \omega^2 \left( \cos \omega + \frac{r}{l} \cos 2\omega t \right) \right] i + [m_A r \omega^2 \cos \omega t] j$$

The shaking force for a multicylinder in line engine;

$$F_s = m_A r \omega^2 \sum_{i=1}^{n} \left[ \cos (\omega t - \phi_i) + \frac{r}{l} \cos 2(\omega t - \phi_i) \right] i$$

Where $n =$ number of cylinder and $\phi_i = 0$
\[ F_s = \cos \omega t \sum_{i=1}^{n} \cos \phi_i + \sin \omega t \sum_{i=1}^{n} \sin \phi_i + \frac{r}{l} \left( \cos 2\omega t \sum_{i=1}^{n} \cos \phi_i \right) \]

<table>
<thead>
<tr>
<th>TABLE 14-1</th>
<th>Force Balance State of a 4-Cylinder Inline Engine with a 0, 90, 180, 270° Crankshaft</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary forces:</td>
<td>[ \sum_{i=1}^{n} \sin \phi_i = 0 \quad \sum_{i=1}^{n} \cos \phi_i = 0 ]</td>
</tr>
<tr>
<td>Secondary forces:</td>
<td>[ \sum_{i=1}^{n} \sin 2\phi_i = 0 \quad \sum_{i=1}^{n} \cos 2\phi_i = 0 ]</td>
</tr>
<tr>
<td>Fourth harmonic forces:</td>
<td>[ \sum_{i=1}^{n} \sin 4\phi_i = 0 \quad \sum_{i=1}^{n} \cos 4\phi_i = 4 ]</td>
</tr>
<tr>
<td>Sixth harmonic forces:</td>
<td>[ \sum_{i=1}^{n} \sin 6\phi_i = 0 \quad \sum_{i=1}^{n} \cos 6\phi_i = 0 ]</td>
</tr>
</tbody>
</table>
The shaking moment

\[ \sum M_L = \sum_{i=1}^{n} z_i F_{S_i} \hat{j} \] (14.6a)

where \( F_{S_i} \) is the shaking force and \( z_i \) is the moment arm of the \( i \)th cylinder. Substituting equation 14.2d (p. 710) for \( F_{S_i} \):

\[ \sum M_L \approx m B r o^2 \left[ \cos \omega t \sum_{i=1}^{n} z_i \cos \phi_i + \sin \omega t \sum_{i=1}^{n} z_i \sin \phi_i \right] + \frac{r}{l} \left[ \cos 2\omega t \sum_{i=1}^{n} z_i \cos 2\phi_i + \sin 2\omega t \sum_{i=1}^{n} z_i \sin 2\phi_i \right] \hat{j} \] (14.6b)

This expression can only be zero for all values of \( \omega t \) if:

\[ \sum_{i=1}^{n} z_i \cos \phi_i = 0 \qquad \sum_{i=1}^{n} z_i \sin \phi_i = 0 \] (14.7a)

\[ \sum_{i=1}^{n} z_i \cos 2\phi_i = 0 \qquad \sum_{i=1}^{n} z_i \sin 2\phi_i = 0 \] (14.7b)

These will guarantee no shaking moments through the second harmonic. We can extend this to higher harmonics as we did for the shaking force.

\[ \sum_{i=1}^{n} z_i \cos 4\phi_i = 0 \qquad \sum_{i=1}^{n} z_i \sin 4\phi_i = 0 \] (14.7c)

\[ \sum_{i=1}^{n} z_i \cos 6\phi_i = 0 \qquad \sum_{i=1}^{n} z_i \sin 6\phi_i = 0 \] (14.7d)
The shaking moment

Crank phase angle
\( \phi_1 = 0^\circ \)
\( \phi_2 = 90^\circ \)
\( \phi_3 = 180^\circ \)
\( \phi_4 = 270^\circ \)

Moment arms of the shaking moment
FIGURE 14.17
Mirror symmetric crankshafts cancel primary moments

(a) Non-symmetric 0, 180, 0, 180° crankshaft
(b) Symmetric 0, 180, 180, 0° crankshaft
The shaking moment

**TABLE 14-2**  Moment Balance State of a 4-Cylinder, Inline Engine with a 0, 90, 180, 270° Crankshaft, and $z_1 = 0$, $z_2 = 1$, $z_3 = 2$, $z_4 = 3$

<table>
<thead>
<tr>
<th>Source</th>
<th>Equation</th>
</tr>
</thead>
</table>
| Primary moments | $\sum_{i=1}^{n} z_i \sin \phi_i = -2$  
$\sum_{i=1}^{n} z_i \cos \phi_i = -2$ |
| Secondary moments | $\sum_{i=1}^{n} z_i \sin 2\phi_i = 0$  
$\sum_{i=1}^{n} z_i \cos 2\phi_i = -2$ |
TABLE 14-3  Force and Moment Balance State of a 4-Cylinder, Inline Engine with a 0, 180, 0, 180° Crankshaft, and $z_1 = 0, z_2 = 1, z_3 = 2, z_4 = 3$

- Primary forces:  
  $$\sum_{i=1}^{n} \sin \phi_i = 0$$  
  $$\sum_{i=1}^{n} \cos \phi_i = 0$$

- Secondary forces:  
  $$\sum_{i=1}^{n} \sin 2\phi_i = 0$$  
  $$\sum_{i=1}^{n} \cos 2\phi_i = 4$$

- Primary moments:  
  $$\sum_{i=1}^{n} z_i \sin \phi_i = 0$$  
  $$\sum_{i=1}^{n} z_i \cos \phi_i = -2$$

- Secondary moments:  
  $$\sum_{i=1}^{n} z_i \sin 2\phi_i = 0$$  
  $$\sum_{i=1}^{n} z_i \cos 2\phi_i = 6$$

(a) Nonsymmetric 0, 180, 0, 180° crankshaft

**FIGURE 14-17**
Mirror-symmetric crankshafts cancel primary moments.
(b) Symmetric 0, 180, 180, 0° crankshaft
The shaking force

The shaking force for a single cylinder in the direction of piston motion $u$ hat with $\theta$ measured from the piston axis is:

$$F_s = m_B r \omega^2 \left( \cos \theta + \frac{r}{l} \cos 2\theta \right) \hat{u}$$

The total shaking force is the vector sum of the contributions from each bank.

$$F_s = F_{SL} + F_{SR}$$
The vee angle \( \nu = 2\gamma \)

Crank angle \( \theta = \omega t - \phi_i \)
The shaking forces for the right (R) and left (L) banks

\[ F_{SR} = m_B r \omega^2 \left( \cos(\theta + \gamma) + \frac{r}{l} \cos^2(\theta + \gamma) \right) \hat{r} \]

\[ F_{SL} = m_B r \omega^2 \left( \cos(\theta - \gamma) + \frac{r}{l} \cos^2(\theta - \gamma) \right) \hat{i} \]

\[
\cos(\theta + \gamma) = \cos \theta \cos \gamma - \sin \theta \sin \gamma \\
\cos(\theta - \gamma) = \cos \theta \cos \gamma + \sin \theta \sin \gamma
\]

\[ F_{SR} = m_B r \omega^2 \left( \cos \theta \cos \gamma - \sin \theta \sin \gamma \right) + \frac{r}{l} (\cos 2 \theta \cos 2 \gamma - \sin^2 \theta)
\]
The shaking forces for the right (R) and left (L) banks

\[ F_{SR} = m_B r \omega^2 \left( \cos \theta \cos \gamma - \sin \theta \sin \gamma \right) + \frac{r}{l} \left( \cos \theta^2 \cos \gamma^2 - \right. \]
\[ F_{SR} = m_B r \omega^2 \left( \cos \theta \cos \gamma - \sin \theta \sin \gamma \right) + \frac{r}{l} \left( \cos \theta \cos 2\gamma - \sin \theta \sin 2\gamma \right) \]

\[ \sum_{i=1}^{n/2} \cos \phi_i \]

\[ + \left( \cos \omega t \sin \gamma + \sin \omega t \cos \gamma \right) \sum_{i=1}^{n/2} \sin \phi_i \]

\[ + \frac{r}{l} \left( \cos 2\omega t \cos 2\gamma - \sin 2\omega t \sin 2\gamma \right) \sum_{i=1}^{n/2} \cos 2\phi_i \]

\[ + \frac{r}{l} \left( \cos 2\omega t \sin 2\gamma + \sin 2\omega t \cos 2\gamma \right) \sum_{i=1}^{n/2} \sin 2\phi_i \]
\[
F_{SL} = m_B r \omega^2 \left( \cos \theta \cos \gamma + \sin \theta \sin \gamma \right) + \frac{r}{l} \left( \cos \theta \cos \gamma \cos \phi \right) + \frac{r}{l} \left( \cos \phi \cos \theta \right)
\]

\[
\sin(\omega t - \phi_i) = \sin \omega t \cos \phi_i - \cos \omega t \sin \phi_i
\]
The shaking force

- A set of sufficient criteria for zero shaking moment through the second harmonic for each bank.

- The shaking force for each bank into components along and normal to central X axis of the vee engine:

\[ F_{s_X} = (F_{s_L} + F_{s_R}) \cos \gamma \hat{i} \]
\[ F_{s_Y} = (F_{s_L} - F_{s_R}) \sin \gamma \hat{j} \]
\[ F_S = F_{s_X} \hat{i} + F_{s_Y} \hat{j} \]
The shaking moment

The moments exist within each bank and their vectors will be orthogonal to the respective cylinder planes.

For the right bank its define a moment unit vector n hat perpendicular to the r hat Z plane.

For the left bank its define a moment unit vector m hat perpendicular to the I hat Z plane.
\[
M_{s_R} \approx m_B r \omega^2 \\
\left( \cos \omega t \cos \gamma - \sin \omega t \sin \gamma \right) \sum_{i=1}^{n/2} z_i \cos \phi_i \\
+ \left( \cos \omega t \sin \gamma + \sin \omega t \cos \gamma \right) \sum_{i=1}^{n/2} z_i \sin \phi_i \\
+ \frac{r}{l} \left( \cos 2\omega t \cos 2\gamma - \sin 2\omega t \sin 2\gamma \right) \sum_{i=1}^{n/2} z_i \cos 2\phi_i
\]

\[
M_{s_L} \approx m_B r \omega^2 \\
\left( \cos \omega t \cos \gamma + \sin \omega t \sin \gamma \right) \sum_{i=n/2+1}^{n} z_i \cos \phi_i \\
- \left( \cos \omega t \sin \gamma - \sin \omega t \cos \gamma \right) \sum_{i=n/2+1}^{n} z_i \sin \phi_i \\
+ \frac{r}{l} \left( \cos 2\omega t \cos 2\gamma + \sin 2\omega t \sin 2\gamma \right) \sum_{i=n/2+1}^{n} z_i \cos 2\phi_i \\
- \frac{r}{l} \left( \cos 2\omega t \sin 2\gamma - \sin 2\omega t \cos 2\gamma \right) \sum_{i=n/2+1}^{n} z_i \sin 2\phi_i
\]
A set of sufficient criteria for zero shaking moment through the second harmonic for each bank. The shaking moment for each bank into components along and normal to central X axis of the vee engine:

\[ M_{sx} = (M_{sl} - M_{sr}) \sin \gamma \]
\[ M_{sy} = (-M_{sl} - M_{sr}) \cos \gamma \]
\[ \mathbf{M}_s = M_{sx} \hat{i} + M_{sy} \hat{j} \]
The inertia torque

- The inertia torques from the right and left banks of a vee engine are:

\[
T_{i21R} \approx \frac{1}{2} m_B r^2 \omega^2 \left[ \frac{r}{2l} \left( \sin(\omega t + \gamma) \sum_{i=1}^{n/2} \cos \phi_i - \cos(\omega t + \gamma) \sum_{i=1}^{n/2} \sin \phi_i \right) - \frac{3r}{2l} \left( \sin 3(\omega t + \gamma) \sum_{i=1}^{n/2} \cos 3\phi_i - \cos 3(\omega t + \gamma) \sum_{i=1}^{n/2} \sin 3\phi_i \right) \right]
\]
\[ T_{i21_L} \approx \frac{1}{2} m_B r^2 \omega^2 \left( \frac{r}{2l} \left( \sin(\omega t - \gamma) \sum_{i=n/2+1}^{n} \cos \phi_i - \cos(\omega t - \gamma) \sum_{i=n/2+1}^{n} \sin \phi_i \right) \right) \]

\[ - \left( \sin 2(\omega t - \gamma) \sum_{i=n/2+1}^{n} \cos 2\phi_i - \cos 2(\omega t - \gamma) \sum_{i=n/2+1}^{n} \sin 2\phi_i \right) \]

\[ - \frac{3r}{2l} \left( \sin 3(\omega t - \gamma) \sum_{i=n/2+1}^{n} \cos 3\phi_i - \cos 3(\omega t - \gamma) \sum_{i=n/2+1}^{n} \sin 3\phi_i \right) \]
For zero inertia torque through the second harmonic in a vee engine it is sufficient that:

\[
\begin{align*}
\sum_{i=1}^{n/2} \sin \phi_i &= 0 \\
\sum_{i=1}^{n/2} \cos \phi_i &= 0 \\
\sum_{i=1}^{n} \sin \phi_i &= 0 \\
\sum_{i=1}^{n} \cos \phi_i &= 0 \\
\sum_{i=1}^{n/2} \sin 2\phi_i &= 0 \\
\sum_{i=1}^{n/2} \cos 2\phi_i &= 0 \\
\sum_{i=1}^{n} \sin 2\phi_i &= 0 \\
\sum_{i=1}^{n} \cos 2\phi_i &= 0 \\
\sum_{i=1}^{n/2} \sin 3\phi_i &= 0 \\
\sum_{i=1}^{n/2} \cos 3\phi_i &= 0 \\
\sum_{i=1}^{n} \sin 3\phi_i &= 0 \\
\sum_{i=1}^{n} \cos 3\phi_i &= 0
\end{align*}
\]
The gas torque is:

\[ T_{g21} \approx F_g r \sum_{i=1}^{n} \left( \sin[\omega t - (\psi_i + \gamma_k)] \left\{ 1 + \frac{r}{l} \cos[\omega t - (\psi_i + \gamma_k)] \right\} \right) \hat{k} \]  \hspace{1cm} (14.13)

where the left bank has a bank angle \( \gamma_k = +\gamma \) and the right bank a bank angle \( \gamma_k = -\gamma \).
Balancing multicylinder engines

- With a sufficient number \((m)\) cylinders, properly arranged in banks of \(n\) cylinders in a multibank engine, an engine can be inherently balanced.

- In two-stroke engine with its crank throws arranged for even firing, all harmonics of shaking force will be balanced except those whose harmonic number is a multiple of \(n\).
Balancing multicylinder engines

- In four-stroke engine with its crank throws arranged for even firing, all harmonics of shaking force will be balanced except those whose harmonic number is a multiple of $n/2$.

- Primary shaking moment components will be balanced if the crankshaft is mirror-symmetric about the central transverse plane.
Balancing multicylinder engines

- For-stroke inline configuration the requires at least six cylinders to inherently balanced through the second harmonic.

- An inline four with a 0°, 180°, 180°, 0° crank shaft has nonzero secondary forces and moments as well as nonzero inertia torque.
Balancing multicylinder engines

The inline six with a mirror-symmetric crank of $0^\circ$, $240^\circ$, $120^\circ$, $120^\circ$, $240^\circ$, $0^\circ$ will have zero shaking forces and moments through the second harmonic, though the inertia torque’s third harmonic will still be present.
A Vee twelve

A Vee twelve is then the smallest vee engine with an inherent state of near perfect balance, as it is two inline sixes on a common crank shaft.

The common vee-eight engine with crankshaft phase angles of 0°, 90°, 270°, 180° has unbalanced primary moment as does the inline four from which is it made.
FIGURE 14-22
Two connecting rods on a common crank throw
Unbalanced shaking forces and shaking moments

(a) Force balancing

(b) Moment balancing
(a) Force balancing
Two types of secondary balancer mechanisms for the four-cylinder inline engine

(a) Lanchester balancer

(b) Nakamura balancer
Problem 14-1

- Draw a crank phase diagram for a three cylinder inline engine with a $0^\circ$, $120^\circ$, $240^\circ$ crank shaft and determine all possible firing orders for
  a) Four stroke cycle
  b) Two stroke cycle

Select the best arrangement to give even firing for each stroke cycle.
The firing order is possible with both two and four stroke design. A 1,2,3 firing order is required for the two-stroke and a 1,3,2 firing order for the four-stroke.

Four stroke cycle

1,3,2
Two stroke cycle

1, 2, 3
Problem 14-2

- Repeat Problem 14-1 for in line four-cylinder engine with 0°, 90°, 270°, 180° crank.
- The firing order is possible with both two and four stroke design.
- A 1,2,3, 4 firing order is required for the two-stroke and a 1,4, 3, 2 firing order for the four-stroke.
Four stroke cycle 1, 4, 3, 2
Two stroke cycle

1, 2, 4, 3
Problem 14-3

- Repeat Problem 14-1 for a 45° vee engine, four cylinder with 0°, 90°, 270°, 180° crank.

- The firing order is possible with both two and four stroke design.

- A 1, 2, 3, 4 firing order is required for the two-stroke and a 1, 4, 3, 2 firing order for the four-stroke.
Four stroke cycle 1, 4, 3, 2
Two stroke cycle

Diagram showing the cycle with phase angles and cylinders:
- Phase Angle: 0, 90, 270, -180
- Cylinders: 1, 2, 3, 4
- TDC markers at various points
- Power blocks at specific crank angles:
  - 180-360 degree range for cylinder 1
  - 360-540 degree range for cylinder 2
  - 540-720 degree range for cylinder 3
  - 720-900 degree range for cylinder 4

Crank Angle: 0, 180, 360, 540, 720
Problem 14-4

- Repeat Problem 14-1 for a 45° vee engine, for two cylinder with 0°, 90°, crank.
- The only possible firing order is 1, 2 for stroke cycle.
Four stroke cycle
Two stroke cycle
Problem 14-5

- Repeat Problem 14-1 for a 90° vee engine, for two cylinder with 0°, 180°, crank.
- Firing order is 1, 2 for either stroke cycle
Four stroke cycle

Diagram showing the phases of the four-stroke cycle, including intake, compression, power, and exhaust phases for each cylinder.
Two stroke cycle

Phase Angle Cyl.
0 1
-180 2

Power
0 180 360 540 720 Crank Angle

TDC

1,2
Problem 14-6

- Repeat Problem 14-1 for a 180° opposed engine, for two cylinder with 0°, 180°, crank.

- Firing order is 1,2 for either stroke cycle
Four stroke cycle
Two stroke cycle

1. Two stroke cycle diagram showing the phase angle and cylinder operation with TDC (Top Dead Center) points and power phases.

2. The diagram illustrates the crank angle in degrees, with power phases occurring at specific intervals.
Problem 14-7

- Repeat Problem 14-1 for a 180° opposed engine, for four-cylinder with 0°, 180°, 180°, 0° crank.

- Firing order is 1,2 for either stroke cycle
Four stroke cycle
Two stroke cycle
THANK YOU