Chapter 8
Induction Machine Modeling Concepts

Electrical drives with induction machines remain the dominant market leader in the field. The combination of a robust low cost squirrel-cage machine, high power density converter, and versatile controller yields a highly adaptable drive for wide ranging rugged industrial applications.

In this chapter, induction machine models are developed which will be used in successive chapters of this book, e.g. with voltage source inverters and field-oriented controls. Initially, a brief review of the induction machine with squirrel-cage rotor is given in terms of a cross-sectional view and simplified symbolic and generic models. A detailed description of the fundamentals can be found in [31, 40, 48], and [68]. As a platform for introducing field-oriented models, first models without leakage inductances are derived showing the essence of torque production of the machine. Central to this chapter is the introduction of a universal flux linkage model which allows a three-to-two inductance transformation leading to a simplified IRTF machine model. This universal model is the stepping stone to the universal field-oriented (UFO) machine model which gives a basic understanding of the transient behavior of induction machines [19]. Furthermore, this model forms the cornerstone for the development of field-oriented control. At the end of this chapter, attention is given to single-phase induction machines. These machines are widely used in domestic appliances and as such it is important to have access to dynamic and steady-state models. A set of tutorials is provided which allows the user to interactively explore the concepts presented in this chapter.

8.1 Induction Machine with Squirrel-Cage Rotor

Figure 8.1 shows the cross-section of a induction machine with a so-called squirrel-cage rotor. The squirrel cage consists of a set of conductors (shown in red), which are short-circuited at both ends by a conductive ring. The cage is
embedded in the rotor lamination as may be observed from Fig. 8.1. A three-phase two-layer winding is housed in the stator of a four-pole machine.

A rotating field created by the stator winding is penetrating the rotor. If the rotor is rotating asynchronously to the stator field (which means it is rotating at a different speed), alternating currents are induced in the squirrel cage. These currents, together with the stator field, are responsible for the torque production of the machine. This is why asynchronous machines are also known as induction machines. If stator field and rotor were rotating synchronously, no currents would be induced and no torque could be produced. Note, that, regardless of the rotor speed, the rotor field and stator field are still rotating synchronously with a phase shift. The difference between the speed of the rotor and the fields is compensated by the frequency of the rotor currents, the so-called rotor slip frequency. The asynchronous nature of the machine is the reason why for the control of induction machines, only the position of stator field and rotor field are required and not the absolute rotor position.

Squirrel cage based induction machines are widely used in industry. However, other configurations include single-phase machines and doubly-fed in-
duction machines. Single-phase machines are discussed in Sect. 8.5. Modeling and control of doubly-fed induction machines can be found in literature [47]. These machines are found in wind turbines and large pump drives. These machines make use of a rotor that is provided with a set of three-phase windings which are connected to a stationary converter (or set of resistors) via a set of slip-rings.

8.2 Zero Leakage Inductance Models of Induction Machines

For the purpose of understanding the basic dynamic and steady-state behavior of the induction machine, it is helpful to initially ignore the presence of stator and rotor leakage inductances. Therefore, symbolic and generic models will be introduced first without leakage inductances showing the essence of torque production of the machine. Furthermore, the zero leakage approach serves as an effective platform for introducing field-oriented models.

8.2.1 IRTF Based Model of the Induction Machine

As a first step in the development of dynamic models for induction machines, it is instructive to consider a simplified IRTF based symbolic concept as given in Fig. 8.2. Using an IRTF means to deploy a dual coordinate reference frame which is linked to the stator and to the rotor of the machine.

Fig. 8.2 Zero leakage IRTF based induction machine model

The mathematical equation set that conforms to the model according to Fig. 8.2 is as follows:
\[ \vec{u}_s = R_s \vec{i}_s + \frac{d\vec{\psi}_m}{dt} \quad (8.1a) \]
\[ \vec{\psi}_m = L_m (\vec{i}_s - \vec{i}_r) \quad (8.1b) \]
\[ 0 = -R_r \dot{\vec{i}}_r + \frac{d\vec{\psi}^{xy}_m}{dt}. \quad (8.1c) \]

Note that the model presented here is very similar to that shown in Fig. 4.13. In the latter case, current excitation was assumed whereas here voltage excitation is imposed for connecting the machine to a voltage source converter. Furthermore, the magnetizing inductance is shown on the other side of the IRTF which can be done with impunity.

The development of a corresponding generic model representation of this simplified two-pole machine model is readily undertaken with the aid of (8.1) and the two IRTF related (4.5) for electromagnetic torque calculation and (4.6) as a load model. An implementation example is given in Fig. 8.3. It makes use of a differentiator module which is unavoidable in the case leakage inductance has been ignored. Note that, for reasons of numerical stability, the use of differentiators should be avoided in a simulation environment whenever possible. The tutorial in Sect. 8.6.1 is based on the generic model presented in this chapter.

![Fig. 8.3 Generic model, zero leakage IRTF based induction motor model](image-url)
8.2 Field-Oriented Model

A suitable starting point for the development of a field-oriented model is the IRTF based machine model shown in Fig. 8.2 and the vector diagram given in Fig. 8.4. The latter diagram shows the stator current and magnetizing flux linkage space vectors $\vec{i}_s$ and $\vec{\psi}_m$ respectively. The latter vector may be represented as $\vec{\psi}_m = \psi_m e^{j\theta_\psi}$, where $\psi_m$ is the amplitude and $\theta_\psi$ the angle between the flux linkage vector and real axis of the stationary reference plane.

Stationary based equations are transformed to the dq plane using $\vec{A} = \vec{A}_dq e^{j\theta}$ with $\omega_s = d\theta/dt$. The conversion process of rotor-oriented equations to a synchronous reference frame, linked to the flux linkage vector $\vec{\psi}_m$, is achieved by a two-step process. Firstly, the rotor based equations are transformed to a stationary reference frame after which the transformation to a synchronous reference frame can be implemented. Equation set (8.1) may be transformed to the dq plane with the aid of the approach outlined above which leads to:

\[\begin{align*}
\vec{u}_s &= R_s \vec{i}_s + \frac{d\vec{\psi}_m}{dt} + j\omega_s \vec{\psi}_m \\ 
\frac{\vec{\psi}_m}{L_m} &= \vec{i}_s - \vec{i}_r \\ 
\frac{d\vec{\psi}_m}{dt} &= R_t \vec{i}_r - j(\omega_s - \omega_m) \vec{\psi}_m.
\end{align*}\]  

(8.2a, 8.2b, 8.2c)

It is noted that in this case some further simplification is possible given that $\vec{\psi}_m = \psi_m$, i.e., the vector is real because the direct axis is aligned with this vector. All the remaining vectors have both a real (direct) and a imaginary (quadrature) component, for example $\vec{i}_s = i_{sd} + j i_{sq}$. Further development of equation set (8.2) in terms of grouping the real components leads to:
\[ u_{sd} = R_s i_{sd} + \frac{d\psi_m}{dt} \quad (8.3a) \]

\[ \frac{\psi_m}{L_m} = i_{sd} - i_{rd} \quad (8.3b) \]

\[ \frac{d\psi_m}{dt} = R_r i_{rd}. \quad (8.3c) \]

Grouping the imaginary components of equation set (8.2) leads to:

\[ u_{sq} = R_s i_{sq} + \omega_s \psi_m \quad (8.4a) \]

\[ i_{rq} = i_{sq} \quad (8.4b) \]

\[ \omega_s \psi_m = R_r i_{rq} + \omega_m \psi_m. \quad (8.4c) \]

The symbolic direct and quadrature model as given in Fig. 8.5 satisfies equation sets (8.3) and (8.4) respectively.

The corresponding generic current-fed induction machine model with currents \( i_{sd} \) and \( i_{sq} \) as input variables, which corresponds to the symbolic field-oriented model, is given in Fig. 8.6.

The model according to Fig. 8.5 and Fig. 8.6 provides some fundamental insights with respect to the operation of the machine. If for example, a current \( i_{sd} \) is applied to the machine at \( t = 0 \), the flux linkage \( \psi_m \) will assume a steady-state value \( \psi_m = L_m i_{sd} \) after a transitional period which is governed by the time constant \( L_m/R_r \). Once the flux linkage reaches its steady-state value, the variables \( d\psi_m/dt \) and \( i_{rd} \) (see Fig. 8.5) will be equal to zero.

The torque \( T_e \) is determined by the product of \( \psi_m \) and \( i_{sq} \) and these components can be controlled independently as is the case for a DC machine.
8.2 Zero Leakage Inductance Models of Induction Machines

Fig. 8.6 Generic representation of current source, field-oriented induction machine model, with zero leakage inductance

with a field winding. In the latter case, the variables $\psi_m$ and $i_{sq}$ are replaced by the field flux linkage $\psi_f$ and armature current $i_a$ respectively. A voltage variable $\omega_s \psi_m$ as shown in Fig. 8.5, is defined by the sum of the voltages $\omega_m \psi_m$ and $R_t i_{sq}$ (cp. (8.4c)). The latter is equal to the product of the slip frequency $\omega_{sl} = \omega_s - \omega_m$ and flux linkage $\psi_m$. If a step increase in the current $i_{sq}$ is made, whilst maintaining a constant $i_{sd}$ value, the torque and slip frequency must also increase stepwise.

**Tutorial Results**

Figure 8.7 shows a typical transient response for a machine in its current form, without a mechanical load, in terms of the torque $T_e$, shaft speed $\omega_m$, slip frequency $\omega_{sl}$, and electrical frequency $\omega_s$. In this example, a current $i_{sd} = 4.0 \, \text{A}$ is applied at $t = 0$ and a quadrature stator current step $i_{sq} = 0 \rightarrow 4.0 \, \text{A}$ is made at $t = 0.8 \, \text{s}$. The results shown are obtained with the model given in the tutorial (see Sect. 8.6.4). The reader is referred to Sect. 8.6.4 for further details of the simulation model.

**Flux/Current Diagram**

Further insight with regard to this type of model can be obtained by considering the flux/current diagram of the machine taken at a particular instance in time. Figure 8.8 shows the flux lines $\phi_d$ linked with the flux linkage $\psi_m$. A current distribution in the stator windings and squirrel-cage rotor is shown for both cases. The current and flux distributions tied to the dq plane rotate at speed $\omega_s$, while the rotor rotates with a shaft speed $\omega_m$.

The direct axis flux distribution (Fig. 8.8(a)) shows the d-axis which is aligned with the flux linkage vector $\psi_m$. The current $i_{sd}$ is shown in distributed form on the stator side. Note that the rotor component shows no current, i.e., $i_{rd} = 0$. The quadrature model, see Fig. 8.8(b), shows the flux
Fig. 8.7 Transient response of field-oriented model with zero leakage inductance

(a) d-axis flux/current diagram

(b) q-axis current diagram

Fig. 8.8 Direct and quadrature flux/current diagram in steady-state
distribution $\phi_d$ and the stator and rotor current distributions which correspond to $i_{sq}$ and $i_{rq}$ respectively. It is noted that the two current distributions are in opposition in the actual machine given, whilst the model assumes the condition $i_{sq} = i_{rq}$. The reason for the supposed discrepancy is linked to the choice of input/output power conventions of the IRTF model. The resultant current distribution in the stator is formed by the two stator components shown in Fig. 8.8.

8.3 Machine Models with Leakage Inductances

In practical machines, not all the magnetic flux linkage is fully coupled between the stator windings and the rotor squirrel-cage. So-called leakage flux paths are present on the stator and rotor side of the machine, which in modeling terms are represented by leakage inductances $L_{\sigma_s}$ and $L_{\sigma_r}$ respectively. In this section, the IRTF and field-oriented modeling approach used in the previous section is extended to accommodate magnetic flux leakage of the machine. In this context, a universal field-oriented (UFO) approach will be introduced, which will prove to be instrumental for the development of a field-oriented controller in the next chapter [16].

8.3.1 Fundamental IRTF Based Model

The simplified model according to Fig. 8.2 is extended to include the rotor and stator based leakage inductances $L_{\sigma_r}$ and $L_{\sigma_s}$ respectively. The rotor leakage inductance has been conveniently relocated to the stator side of the IRTF module to form a three-element circuit network which consists of the two leakage inductances and the magnetizing inductance $L_m$. Note that the use of an IRTF module allows the positioning of an inductance to either side without having to change its value, and that relocating the leakage inductance does not affect the torque $T_e$, as seen in Sect. 4.1.

Fig. 8.9 Three inductance, IRTF based induction machine model
The equation set which corresponds to Fig. 8.9 is as follows:

\[
\vec{u}_s = R_s \vec{i}_s + \frac{d\vec{\psi}_s}{dt} \quad (8.5a)
\]

\[
\vec{\psi}_s = \vec{\psi}_m + L_{\sigma s} \vec{i}_s \quad (8.5b)
\]

\[
\vec{\psi}_r = \vec{\psi}_m - L_{\sigma r} \vec{i}_r \quad (8.5c)
\]

\[
\vec{\psi}_m = L_m (\vec{i}_s - \vec{i}_r) \quad (8.5d)
\]

\[
0 = -R_r \vec{i}_{xy} + \frac{d\vec{\psi}_{xy}}{dt} \quad (8.5e)
\]

On the basis of the symbolic model given in Fig. 8.9 and equation set (8.5), a generic IRTF based symbolic model can be developed. However, particular attention must be given in terms of its numerical implementation to avoid algebraic loops. The cause of this problem is the presence of two leakage inductance circuit elements which may be avoided as will become apparent in the following section. Furthermore, it is difficult to determine individual values for these two inductances in a squirrel-cage motor. The reason for this is that these values are usually determined by a locked rotor test [68] which yields an estimate for the combined leakage inductance \(L_{\sigma s} + L_{\sigma r}\). The combined leakage inductance value is then (usually) arbitrarily divided by two in order to arrive at values for the individual elements. As a result, the rotor leakage inductance is measured at line frequency (50 Hz or 60 Hz). Due to rotor deep bar and saturation effects [13, 42], the rotor leakage inductance value can deviate strongly when operating under field-oriented control.

### 8.3.2 Universal IRTF Based Model

The IRTF model according to Fig. 8.9 can be transformed to a so-called universal three-inductance configuration which makes use of a transformation coefficient \(a\). By changing the value of this parameter, the user is able to alter the model from, for example, a three- to two-inductance type model where the circuit element which represents the leakage inductance can be located on either side of the equivalent magnetizing inductance component. The term universal reflects the flexibility of this new model in terms of being able to change the inductance parameters simultaneously without affecting the no-load or short circuit impedance of the original inductance network. It will be shown in Sect. 8.3.4 that such a transformation capability is of importance for the development of a so-called universal field-oriented (UFO) machine model [16].
Parameter Definition for the Universal Model

The aim of this section is therefore to define a set of inductance parameters $L_M$, $L_{\sigma S}$, and $L_{\sigma R}$ for a revised symbolic machine model as given in Fig. 8.10 which is able to replace the three-element inductance network of the original model (see Fig. 8.9). The impedance as viewed from either side of the revised inductance network must correspond to the values found in the original inductance network and should not be affected by changes in the transformation factor $a$. In order to achieve this aim, an ITF module with transformation ratio $a:1$ is introduced in the new model.

Fig. 8.10 Universal, IRTF based induction machine model, with ITF module

The transformation process is initiated by considering equation set (8.5) which is linked to the model given in Fig. 8.9. In particular, it is helpful to consider (8.5b) and (8.5d) which may also be written as

$$\vec{\psi}_s = L_s \vec{i}_s - L_m \vec{i}_r - a L_m \vec{i}_s + a L_m \vec{i}_s$$  \hspace{1cm} (8.6)

with $L_s = L_m + L_{\sigma s}$. This expression may then be written as

$$\vec{\psi}_s = \underbrace{(L_s - a L_m)}_{L_{\sigma s}} \vec{i}_s + a L_m \underbrace{(\vec{i}_s - \vec{i}_R)}_{L_{\sigma M}}$$  \hspace{1cm} (8.7)

where the parameters $L_{\sigma S}$ and $L_M$ are introduced, representing a generalized leakage inductance and magnetizing inductance. Furthermore, a scaled rotor current vector $\vec{i}_R$ is introduced in (8.7) which along with the scaled rotor flux linkage vector $\vec{\psi}_R$ is defined as

$$\vec{i}_R = \frac{\vec{i}_r}{a}$$  \hspace{1cm} (8.8a)

$$\vec{\psi}_R = a \vec{\psi}_r.$$  \hspace{1cm} (8.8b)

The scaled rotor flux linkage vector $\vec{\psi}_R$ represents the scaled (by the transformation factor $a$) rotor flux linkage vector $\vec{\psi}_r$. The choice of scaling for $\vec{i}_R$ and $\vec{\psi}_R$ is such that the product of the current and flux linkage vectors as well as the impedance remain unaffected by the scaling. In the universal model
(see Fig. 8.10) (8.8) is represented by the ITF module with winding ratio $a : 1$. Equations (8.8b) and (8.5c) form the basis for the second part of the proposed model transformation. Use of these two equations to represent the scaled rotor flux linkage vector $\vec{\psi}_R$ gives

$$\vec{\psi}_R = a L_m \vec{i}_s - a^2 L_r \vec{i}_R - a L_m \vec{i}_R + a L_m \vec{i}_R$$

(8.9)

with $L_r = L_m + L_{sr}$. This expression may also be rewritten as

$$\vec{\psi}_R = a L_m \left( \vec{i}_s - \vec{i}_R \right) - \left( a^2 L_r - a L_m \right) \vec{i}_R$$

(8.10)

where a second leakage inductance parameter $L_{sr}$ is introduced. The resultant flux linkage vector based equation set as given by (8.7) and (8.10) may also be written as

$$\vec{\psi}_s = L_{ss} \vec{i}_s + L_M \vec{i}_M$$

(8.11a)

$$\vec{\psi}_R = L_M \vec{i}_M - L_{sr} \vec{i}_R$$

(8.11b)

where $\vec{i}_M = \vec{i}_s - \vec{i}_R$ represents the scaled magnetizing current vector. The flux linkage equation set (8.11) contains a set of leakage inductances and magnetizing inductance which are a function of the transformation variable $a$. This new set on inductances is conveniently summarized in equation set (8.12):

$$L_{ss} = L_m \left( \frac{L_s}{L_m} - a \right)$$

(8.12a)

$$L_{sr} = a L_r \left( a - \frac{L_m}{L_r} \right)$$

(8.12b)

$$L_M = a L_m$$

(8.12c)

Observation of equation set (8.12) shows that if the transformation variable $a$ is bound by the condition

$$\frac{L_m}{L_r} \leq a \leq \frac{L_m}{L_s}$$

(8.13)

then the leakage inductances $L_{ss}$ and $L_{sr}$ remain greater than or equal to zero.

The variation of $a$ as defined by (8.13) is relatively small, hence it is helpful to introduce a percentage transformation coefficient $\Gamma_a$ which is defined over the range $\pm 100\%$, with $\Gamma_a = 0\%$ set to correspond to a universal model with $a = 1$. The relationship between $\Gamma_a$ and $a$ may be written as
8.3 Machine Models with Leakage Inductances

\[
\begin{align*}
\text{if } \Gamma_a \geq 0 &: & a &= 1 + \frac{\Gamma_a}{100} \left( \frac{L_s}{L_m} - 1 \right) \quad (8.14a) \\
\text{if } \Gamma_a < 0 &: & a &= 1 + \frac{\Gamma_a}{100} \left( 1 - \frac{L_m}{L_r} \right) \quad (8.14b)
\end{align*}
\]

Hence, a value of \(\Gamma_a = 100\%\) corresponds to a universal model with \(a = L_s/L_m\) and \(L_{\sigma S} = 0\), conversely setting \(\Gamma_a = -100\%\) gives \(a = L_m/L_r\) and \(L_{\sigma R} = 0\).

**Vector Representation of the Universal Model**

It is instructive to consider the model transformation from a graphical perspective. This may be achieved by considering the flux linkage/current vector (8.5b) and (8.5d) for the original model and representing these in a vector diagram for an arbitrarily chosen set of currents \(\vec{i}_s\) and \(\vec{i}_r\) and a set of inductances \(L_m, L_{\sigma s},\) and \(L_{\sigma r}\).

![Vector diagrams](image)

(a) Vector diagram for original model (figure 8.9)  
(b) Vector diagram for universal model with \(\Gamma_a = 50\%\)

**Fig. 8.11** Comparison of vector diagrams for original and universal induction machine model

An example of such a vector plot is shown in Fig. 8.11(a). A similar exercise can also be undertaken for the universal model which requires access to the flux linkage/current (8.11a) and (8.11b). In this case, the transformation variable must be given a specific value in order to derive the vector plot which corresponds to the current vectors and inductances chosen for the original model. The universal model-based vector diagram as given in Fig. 8.11(b) is shown with a transformation variable of \(\Gamma_a = 50\%\). Figure 8.11(a) and Fig. 8.11(b) show an \(a\)-axis which is a line defined by the endpoints of the vectors \(\vec{\psi}_s\) and \(\vec{\psi}_m\) respectively. This line is significant because it represents the locus of the vector \(\vec{\psi}_M\) endpoint as function of the transformation variable \(a\) [16]. For any user defined value of \(a\) the transformation defines the
location of the vector \( \vec{\psi}_M \) and corresponding vectors \( \vec{\psi}_R \) and \( \vec{i}_R \), whereby
the latter two vectors are only changed (with respect to the vectors \( \vec{\psi}_r \) and
\( \vec{i}_r \)) in terms of their magnitude (see (8.8)), as may also be observed from
Fig. 8.11(b).

Symbolic Representation of the Universal Model
The universal model in its present form makes use of an ITF module to trans-
form the vectors \( \vec{\psi}_R \) and \( \vec{i}_R \) to their original values \( \vec{\psi}_r \) and \( \vec{i}_r \). It is from
a modeling perspective not strictly necessary to undertake such a transforma-
tion provided that the user takes into account the fact that a scaling of these
rotor based variables takes place which is dependent on the choice of trans-
formation factor \( a \). The ITF module may be omitted by relocating the IRTF
and rotor resistance \( R_t \) to the primary side of the ITF module. Relocating
the IRTF module will not affect the torque but the referred rotor resistance
\( R_R \) must be calculated using

\[
R_R = a^2 R_t. \tag{8.15}
\]

The resultant universal IRTF based symbolic machine model is shown in
Fig. 8.12.

![Fig. 8.12 Universal, IRTF based induction machine model](image)

The corresponding equation set for the universal IRTF based model is of the
form:

\[
\begin{align*}
\vec{u}_s &= R_s \vec{i}_s + \frac{d\vec{\psi}_s}{dt} \tag{8.16a} \\
\vec{\psi}_s &= \vec{\psi}_M + L_{\sigma S} \vec{i}_s \tag{8.16b} \\
\vec{\psi}_R &= \vec{\psi}_M - L_{\sigma R} \vec{i}_R \tag{8.16c} \\
\frac{\vec{\psi}_M}{L_M} &= \vec{i}_s - \vec{i}_R \tag{8.16d} \\
0 &= -R_R \vec{i}_R^x + \frac{d\vec{\psi}_R^x}{dt}. \tag{8.16e}
\end{align*}
\]
The significance of the universal model transformation can be demonstrated by considering the following three values (of which two are chosen at opposite ends of the scale, see (8.13)) for the transformation variable $a$, namely:

- $a = \frac{L_m}{L_\sigma} (\Gamma_a = -100\%)$: Under these conditions, the model shown in Fig. 8.12 is reduced to two inductances $L_{\sigma S}$ and $L_M$, i.e., $L_{\sigma R} = 0$. Furthermore, the vector $\tilde{\psi}_M$ equals $\tilde{\psi}_R$ as may also be observed with the aid of Fig. 8.11(b). This model will be referred to the rotor flux based IRTF model.

- $a = 1 (\Gamma_a = 0\%)$ The universal model is reduced to the original five-parameter model as given in Fig. 8.9.

- $a = \frac{L_s}{L_m} (\Gamma_a = 100\%)$ The universal model according to Fig. 8.12 is reduced to an alternative two-inductance model with $L_{\sigma S} = 0$. Furthermore, the vector $\tilde{\psi}_M$ is equal to $\tilde{\psi}_s$ under these circumstances as may also be observed with the aid of Fig. 8.11(b). This model will be referred to the stator flux based IRTF model.

**Generic Representation of the Universal Model**

A generic representation of the symbolic model as given in Fig. 8.13 may be developed with the aid of the terminal (8.16a) and (8.16e).

The model in question makes use of a generic module identified by the name $L^{-1}$ which represents the matrix $[L^{-1}]$ defined by (8.17).

![Fig. 8.13](image-url)
\[
\begin{bmatrix}
\vec{i}_s \\
\vec{i}_R
\end{bmatrix} = \frac{1}{L_s L_R - (L_M)^2} \begin{bmatrix}
L_R - L_M \\
L_M - L_s
\end{bmatrix} \begin{bmatrix}
\hat{\vec{\psi}}_s \\
\hat{\vec{\psi}}_R
\end{bmatrix}
\]

(8.17)

with \( L_s = L_M + L_{\sigma S} \) and \( L_R = L_M + L_{\sigma R} \), where \( L_M \), \( L_{\sigma S} \), and \( L_{\sigma R} \) are defined by (8.12). Expression (8.17) is found with the aid of (8.11) in which the vector \( \hat{\vec{i}}_M \) is redefined in terms of the current vectors \( \vec{i}_s \) and \( \vec{i}_R \). The model according to Fig. 8.13 does not generate the vector \( \vec{\psi}_M \) explicitly, instead this variable can be calculated using (8.16c). It is emphasized that this generic model can be used for any transformation factor \( a \) within the range defined by (8.13) without encountering any algebraic loops in the simulation. The reader is reminded of the fact that changes in the transformation variable \( a \) affect the matrix \( L^{-1} \) terms as well as the variable \( R_R \). In some cases, it is beneficial to represent (8.17) in terms of the variables \( L_m, L_{\sigma S}, \) and \( L_{\sigma R} \), which represent the inductances of the original model (see Fig. 8.9). Use of (8.12) with (8.17) leads to

\[
\begin{bmatrix}
\hat{\vec{i}}_s \\
\hat{\vec{i}}_R
\end{bmatrix} = \frac{1}{\sigma_u L_s} \begin{bmatrix}
1 \\
\frac{1}{a}\left(\frac{L_m}{L_s L_{\sigma R}}\right)
\end{bmatrix} \begin{bmatrix}
\hat{\vec{\psi}}_s \\
\hat{\vec{\psi}}_R
\end{bmatrix}
\]

(8.18)

where \( \sigma_u = 1 - L_m^2/L_s L_{\sigma R} \) represents the leakage factor [16, 40, 48], which is a machine characteristic and not a function of the transformation variable \( a \). The tutorial given in Sect. 8.6.2 is directly based on the generic model (see Fig. 8.13) and demonstrates the use of the universal transformation concept as discussed in this section.

### 8.3.2.1 Rotor Flux Based IRTF Model

Use of the transformation variable \( a = L_m/L_r \) (\( \Gamma_a = -100\% \)) with the model according to Fig. 8.12 leads to the symbolic model shown in Fig. 8.14.

![Fig. 8.14](image-url) IRTF based induction machine model with \( a = L_m/L_r \).
The corresponding equation set for the rotor flux based IRTF based model is of the form

\[ \vec{u}_s = R_s \vec{i}_s + \frac{d\vec{\psi}_s}{dt} \]  
\[ \vec{\psi}_s = \vec{\psi}_R + L_{\sigma S} \vec{i}_s \]  
\[ \frac{\psi_{xy}^R}{L_M} = \vec{i}_s - \vec{i}_{xy} \]  
\[ 0 = -R_R \vec{i}_R + \frac{d\vec{\psi}_{xy}^R}{dt}. \]

The symbolic model according to Fig. 8.14 will be used for representing the standard induction machine. The generic dynamic model as given in Fig. 8.15 corresponds to this symbolic model and equation set (8.19). It is emphasized that the rotor flux based IRTF model is able to accommodate dynamic as well as steady-state operation. However, for the latter a phasor type analysis may be more convenient as will be shown in Sect. 8.3.6.

![Fig. 8.15 Four-parameter, IRTF based induction motor model](image_url)

Note that the IRTF model can be readily extended to include (among others) homopolar effects, rotor skin effect, however such models are not included here as these are outside the scope of this book.

**Tutorial Results**

In Sect. 8.6.7, a simulation example is given which is based on the model described above. In this tutorial, the model is used to examine the line start of a 22 kW four-pole delta-connected induction machine. An exam-
ple of the results obtained with this simulation model, as indicated in Fig. 8.16, shows the machine shaft torque, shaft speed, and line current over a 2 s start up sequence, where a load torque step is applied at $t = 1 \text{s}$. The reader is referred to the tutorial given in Sect. 8.6.7 for further details and the opportunity to interactively examine this model concept.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig816.png}
\caption{Line start simulation of a 22 kW delta connected machine, showing shaft torque, speed and line current}
\end{figure}

8.3.2.2 Stator Flux Based IRTF Model

Setting the transformation variable to $a = L_m / L_r$ ($I_a = 100\%$) reduces the symbolic model given in Fig. 8.12 to the form shown in Fig. 8.17 [71]. This model is instructive because it shows how the rotor current is affected by the series impedance formed by the leakage inductance $L_{\sigma R}$ and rotor resistance $R_R$.

The corresponding equation set for the stator flux based IRTF model may be found by making use of equation set (8.16) with $L_{\sigma S} = 0$, $L_M = L_S$, and $\vec{\psi}_M = \vec{\psi}_s$, which gives
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Fig. 8.17 IRTF based induction machine model with $a = L_s / L_m$

\[
\begin{align*}
\vec{u}_s &= R_s \vec{i}_s + \frac{d\vec{\psi}_s}{dt} & (8.20a) \\
\vec{\psi}_s &= \vec{\psi}_R + L_{\sigma R} \vec{i}_s & (8.20b) \\
\vec{\psi}_s &= \vec{i}_s - \vec{i}_R & (8.20c) \\
0 &= -R_R \vec{i}_R + \frac{d\vec{\psi}_R}{dt}. & (8.20d)
\end{align*}
\]

8.3.3 Universal Stationary Frame Oriented Model

The IRTF based models introduced in Sect. 8.3.2 have components which are linked to vectors in a stationary as well as a shaft-oriented reference frame. To simplify analysis, a model is derived where all voltage, current, and flux linkage vectors are linked to a common stationary reference frame. To realize this aim, the rotor coordinate based (8.16e) must be converted to stationary coordinates. The general space vector conversion required for this task is of the form $\vec{A} = \vec{A}^{xy} e^{j\theta}$ with $\theta = \omega_m t$. The revised rotor based equation in stationary coordinates is of the form

\[
0 = -R_R \vec{i}_R + \frac{d\vec{\psi}_R}{dt} - j\omega_m \vec{\psi}_R. 
\]

(8.21)

Use of (8.21) and the stationary frame oriented elements of equation set (8.20) leads to the symbolic model given in Fig. 8.18 which appears in the drives literature with different values for the transformation variable $a$. Note that the conversion process used here is convenient when dealing with steady state sinusoidal supplies. In this case, the space vectors that can be readily transformed to stationary phasor diagrams.

The corresponding generic model representation as shown in Fig. 8.19 makes use of the generic module $L^{-1}$ introduced in the Sect. 8.3.2. A tutorial which is based on the generic model given here is discussed in Sect. 8.6.3.
8.3.4 Universal Field-Oriented (UFO) Model

In the machine models discussed previously, the current, voltage, and flux linkage space vectors were defined with respect to a stationary and/or shaft-oriented reference frame. In this section, a so-called universal field-oriented (UFO) transformation is introduced where the stator and rotor based space equations are tied to the flux linkage vector $\vec{\psi}_M$.

This approach combines the advantages of a universal inductance model, as discussed in Sect. 8.3.2, with a so-called field-oriented transformation that leads to synchronous model representation. The development of UFO based models in this section is particularly instructive for the development of field-oriented control concepts in Chap. 9. In this context, the models to be dis-
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Discussed are current-excited because vector-controlled drives often utilize some form of current control as discussed in Chap. 3.

**Development of a Symbolic UFO Model**

The development of a UFO type model is based on the field-oriented (synchronous) reference frame with a so-called *direct* and *quadrature* axis, i.e., \( \bar{x}^{dq} = x_d + jx_q \). The direct axis is aligned with flux linkage vector \( \bar{\psi}_M \), hence \( \psi_{Md} = \psi_M \) and \( \psi_{Mq} = 0 \), as can be observed in Fig. 8.20.

![Diagram showing direct and quadrature axis](image)

**Fig. 8.20** Vector diagram with direct and quadrature axis

The approach required to derive the symbolic and generic model of the UFO model with leakage inductances \( L_{\sigma S} \) and \( L_{\sigma R} \) (see Fig. 8.12) is similar to the method described for the zero leakage case. Consequently, the coordinate transformation process may be undertaken with the aid of (8.16) and (8.21). However, in this case the stator and rotor flux linkage space vectors \( \bar{\psi}_s \) and \( \bar{\psi}_R \) must in the course of this transformation be expressed in terms of the magnetizing vector \( \bar{\psi}_M \), given that the d-axis of synchronous reference frame is aligned with this variable (as shown in Fig. 8.20). The equation set for the generalized UFO based model may be written as

\[
\begin{align*}
\dot{\bar{u}}^{dq}_s &= R_s \bar{i}^{dq}_s + \frac{d}{dt} \bar{\psi}^{dq}_s + j\omega_s \bar{\psi}^{dq}_s \\
\bar{\psi}^{dq}_s &= \psi_M + L_{\sigma S} \bar{i}^{dq}_s \\
\bar{\psi}^{dq}_R &= \psi_M - L_{\sigma R} \bar{i}^{dq}_R \\
\frac{\psi_M}{L_M} &= \bar{i}^{dq}_s - \bar{i}^{dq}_R
\end{align*}
\]  

(8.22a, 8.22b, 8.22c, 8.22d)
The symbolic direct and quadrature models, as shown in Fig. 8.21, are found by rearranging equation set (8.22) and by grouping the real and imaginary terms. The real terms for the direct axis model are:

\[
u_{sd} = R_s i_{sd} - \omega_s L_{\sigma S} i_{sq} + L_{\sigma S} \frac{di_{sd}}{dt} + \frac{d\psi_M}{dt} \tag{8.23a}
\]

\[
\frac{\psi_M}{L_M} = i_{sd} - i_{Rd} \tag{8.23b}
\]

\[
\frac{d\psi_M}{dt} = L_{\sigma R} \frac{di_{Rd}}{dt} - (\omega_s - \omega_m) L_{\sigma R} i_{Rq} + R_R i_{Rd}. \tag{8.23c}
\]

The imaginary terms for the quadrature axis model form the following equation set:

\[
u_{sq} = R_s i_{sq} + L_{\sigma S} \frac{di_{sq}}{dt} + \omega_s L_{\sigma S} i_{sd} + e_q \tag{8.24a}
\]

\[
i_{sq} = i_{Rq} \tag{8.24b}
\]

\[
e_q = L_{\sigma R} \frac{di_{Rd}}{dt} + (\omega_s - \omega_m) L_{\sigma R} i_{Rd} + R_R i_{Rq} + \omega_m \psi_M. \tag{8.24c}
\]

The impact of including the leakage inductance components of the machine in the symbolic direct and quadrature models is significant as may be observed by comparing Fig. 8.21 with the zero leakage model in Fig. 8.5. The model complexity increases as the rotor current vector \( \vec{i}_{Rdq} \) is no longer perpendicular to the main magnetizing flux vector \( \vec{\psi}_m \).

Development of a Generic UFO Model

The development of a generic stator current model which corresponds to Fig. 8.21 may be undertaken with the aid of (8.23c) and (8.24c), and replacing the variables \( i_{Rd} \) and \( i_{Rq} \) with the variables \( i_{sd} \), \( i_{sq} \), and \( \psi_M \) as defined by (8.23b) and (8.24b). Subsequent mathematical manipulation gives

\[
\frac{d}{dt} \left( \frac{L_R}{L_M} \psi_M \right) + \frac{R_R}{L_M} \psi_M = R_R i_{sd} + L_{\sigma R} \frac{di_{sd}}{dt} - \omega_{sl} L_{\sigma r} i_{sq} \tag{8.25a}
\]

\[
\omega_{sl} \left( \frac{L_R}{L_M} \psi_M - L_{\sigma R} i_{sd} \right) = L_{\sigma R} \frac{di_{sq}}{dt} + R_R i_{sq} \tag{8.25b}
\]

with \( L_R = L_{\sigma R} + L_M \) and slip frequency \( \omega_{sl} = (\omega_s - \omega_m) \) as defined previously. Equation (8.25a) yields the basic set of generic modules which are needed to generate the flux linkage variable \( \psi_M \) with inputs \( i_{sd} \), \( i_{sq} \), and \( \omega_{sl} \). However, to complete this model the slip frequency must also be expressed in terms of the input variables \( i_{sd}, i_{sq}, \) and \( \psi_M \). The latter can readily be achieved by rewriting (8.25b) as follows:
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Fig. 8.21 Symbolic UFO model: direct/quadrature axis configuration, with leakage inductance

\[ \omega_s L_{\sigma S} i_{sd} \]

\[ \omega_s L_{\sigma S} i_{sq} \]

\[ (\omega_s - \omega_m) L_{\sigma R} i_{Rd} \]

\[ (\omega_s - \omega_m) L_{\sigma R} i_{Rq} \]

\[ e_d = \frac{d\psi_M}{dt} \]

\[ e_q = \omega_s \psi_M \]

\[ \omega_{sl} = \frac{L_{\sigma R} \frac{d i_{sq}}{dt} + R_R i_{sq}}{\frac{L_R}{L_M} \psi_M - L_{\sigma R} i_{sd}}. \]  

Equations (8.25) and (8.26) together with the torque equation \( T_e = \psi_M i_{sq} \), define the complete generic direct/quadrature model of the UFO machine concept shown in Fig. 8.22. Some indication with respect to its functioning is possible at this stage by considering this model without rotor leakage, in which case the parameter \( L_{\sigma R} \) should be set to zero. The model according to Fig. 8.22 is particularly useful for considering two special cases where the d-axis and corresponding \( \vec{\psi}_M \) is aligned with either the rotor flux linkage vector \( \vec{\psi}_R \) or stator flux linkage vector \( \vec{\psi}_s \). Both cases will be discussed in the next two subsections.

8.3.4.1 Rotor Flux Oriented Model

If a transformation value of \( a = L_m/L_r \) is used in the generalized UFO model, a d-axis alignment with the rotor flux linkage vector \( \vec{\psi}_R \) takes place as may be observed from Fig. 8.23. This vector diagram shows the spatial orientation of the current/flux linkage vectors for rotor flux oriented models. As such, this figure is a specific case of the more generalized case shown in Fig. 8.20.

With the present choice of transformation value, the symbolic direct and quadrature models shown in Fig. 8.21 convert to the form indicated in
Fig. 8.22 Generic direct and quadrature UFO based, stator current source induction machine model

Fig. 8.23 Rotor flux oriented vector diagram
The reason for this is that the leakage inductance $L_{\sigma R}$ will be zero for the selected $a$ value (see $(8.12b)$).

![Figure 8.24](image.png)

**Fig. 8.24** Symbolic rotor flux oriented model: direct and quadrature axis topologies

The implications for the generic model which is linked with the choice of $dq$ coordinates may be observed with the aid of Fig. 8.25. This figure is based on Fig. 8.22 where all $L_{\sigma R}$ related terms are omitted as $L_{\sigma R} = 0$ for rotor flux orientation ($a = L_m / L_r$). Under these circumstances, the model reduces to the form given by Fig. 8.6 (where here, due to the choice of $a$, $R_R$ replaces $R_r$ and $L_R$ replaces $L_m$), which represents the zero rotor leakage model. The latter is perhaps not surprising given that the model assumes current excitation, which implies, as may be observed from the symbolic model in Fig. 8.24, that only the voltages $e_d$ and $e_q$ are affected by the parameters and variables to the right of these diagrams. An important benefit of a rotor-oriented flux model is that there is a complete decoupling between the direct and quadrature currents. This implies that a change in torque may be undertaken by changing the quadrature current only as may be observed from Fig. 8.6. Likewise, a change in direct current will only affect the flux linkage magnitude $\psi_M = \psi_R$. Given the simplicity of these models, they were used first by the inventors of field-oriented control in a time when no digital implementation of the algorithm was possible [7, 27]. A model representation with a single leakage inductance $L_{\sigma S}$ on the stator side of the magnetizing inductance $L_M$ is often used.

**Tutorial Results**

The tutorial given in Sect. 8.6.5 is based on the UFO generic model concept shown in Fig. 8.22. The current excitation and parameters $R_r$ and $L_m$ used
here, to derive the results shown in Fig. 8.26 are the same as those used to
derive the zero leakage results given in Fig. 8.7. However, the model has been
extended to accommodate the leakage inductance parameters $L_{\sigma s}$ and $L_{\sigma r}$
and the transformation variable $a$ is introduced. Variable $a$ may be varied
to allow the model to be used in any UFO reference. Here, the model is
set to a rotor flux oriented reference frame, and the transient results shown
in Fig. 8.26 underline the fact that the direct and quadrature models are
decoupled, given that a change in the torque does not affect the flux linkage $\psi_M$.

A qualitative comparison of the results shown in Fig. 8.7 and Fig. 8.26
demonstrates that the results are similar. This implies that the use of a
rotor flux oriented model leads to a transient response which matches that
of a machine without leakage inductance. It is precisely the combination of
using direct/quadrature current excitation and setting the leakage inductance
parameter $L_{\sigma R}$ to zero which leads to a decoupled model that may be readily
used for realizing field-oriented control in a rotor flux oriented reference frame.

### 8.3.4.2 Stator Flux Oriented Model

If the transformation variable is set to $a = L_s / L_m$, the dq symbolic model
is reduced from the configuration shown in Fig. 8.21 to that indicated in
Fig. 8.27. The reason for this is that the leakage inductance $L_{\sigma S}$ will be zero
for this $a$ value (see (8.12b)).

The corresponding vector diagram for this model representation as shown
in Fig. 8.28 highlights the fact that the flux linkage vector $\vec{\psi}_M$ (and there-
fore the d-axis) is now aligned with the vector $\vec{\psi}_s$. A further observation of
Fig. 8.26 Transient response of rotor flux oriented UFO model

Fig. 8.27 Symbolic stator flux oriented model: direct/quadrature axis configurations
Fig. 8.28 shows that the magnitudes of the vectors $\vec{\psi}_R$ and $\vec{i}_R$ are changed. However, their spatial orientation with respect to the vectors $\vec{\psi}_r$ and $\vec{i}_r$ remains unaffected. In contrast to the rotor flux oriented symbolic models, the direct and quadrature models exhibit a degree of cross-coupling which is dictated by the leakage inductance $L_{\sigma R}$.

**Tutorial Results**

The tutorial (see Sect. 8.6.5) used in the previous section for the presentation of machine transient results may be conveniently used in this section, provided that the transformation value is set to $a = L_s/L_m$. With this choice of transformation variable, the UFO based simulation model will operate with a stator flux oriented reference frame. The current excitation and machine parameters used to derive the results given in Fig. 8.26 are also applied to the numerical results given in Fig. 8.29.

A qualitative comparison between the results shown in Fig. 8.29 and Fig. 8.26 demonstrates that they are markedly different. For example, a Dirac type response occurs in the slip frequency $\omega_{sl}$ which in turn appears in the electrical frequency $\omega_s$. The cause of this phenomenon is the step change in the quadrature current which leads to an instantaneous change of the flux linkage vector $\vec{\psi}_s$ with respect the vector $\vec{\psi}_r$. In addition to this Dirac function, a step in the slip frequency occurs which leads to a change in the flux linkage $\psi_M = \vec{\psi}_s$. The results given in Fig. 8.29 confirm the presence of coupling between the direct and quadrature models and the impact of a torque step on the instantaneous slip frequency. Inverting the model in question for the purpose of developing stator flux oriented control is therefore a more demanding task as will become apparent in Chap. 9.
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8.3.5 Synchronous Frame Oriented Heyland diagram

As mentioned earlier, most drives assume some type of direct/quadrature axis current control, where the flux linkage $\psi_M$ is maintained at a constant value. It is therefore of interest to consider the quasi-stationary stator current $\vec{i}_{ds}^q$ locus under varying slip conditions and constant flux linkage $\psi_M$. This analysis may be undertaken by considering the steady-state form of (8.23c) and (8.24c) which upon elimination of $\omega_{sl} = (\omega_s - \omega_m)$ and after some mathematical manipulation leads to:

$$
\left( i_{sd} - \frac{\psi_M}{2} \left( \frac{1}{L_{\sigma R}} + \frac{1}{L_M} \right) \right)^2 + i_{sq}^2 = \frac{\psi_M^2}{4} \left( \frac{1}{L_{\sigma R}} - \frac{1}{L_M} \right)^2.
$$

This expression represents a circle in the complex dq plane with its center at coordinates $(\psi_M/2(1/L_{\sigma R} + 1/L_M), 0)$ and radius $\psi_M/2(1/L_{\sigma R} - 1/L_M)$ as indicated in Fig. 8.30(a). This type of current locus diagram, known as a Heyland diagram [28, 29, 36] shows the interaction between the stator current vector $\vec{i}_s$ (in this case represented in synchronous coordinates), the torque $T_e$ and the slip frequency $\omega_{sl}$. The circle shows that a given $\psi_M$ value corresponds to a maximum quadrature current and maximum torque $T_e^{max}$ value.
In practice, this maximum quadrature current value is normally outside the rated value, given that $L_{\sigma R} \ll L_M$. The influence of the transformation variable is clearly apparent in Fig. 8.30(b) given the dependency of the latter on the parameters $L_{\sigma R}$ and $L_M$ (see (8.12)). When this transformation ratio is changed from $L_s/L_m \rightarrow L_m/L_r$, the radius increases to infinity, given the variation of the leakage inductance from $L_{\sigma R} \rightarrow 0$. This implies that the current locus at $a = L_m/L_r$ is reduced to a straight line as shown in Fig. 8.30(a).

**8.3.6 Steady-State Analysis of Voltage-Source-Connected Induction Machines**

The steady-state characteristics of the induction machine are studied with the aid of Fig. 8.14, where it is assumed that the stator is connected to a three-phase sinusoidal supply which is represented by the space vector $\hat{u}_s = \hat{u}_s e^{j\omega_s t}$. The basic characteristics of the machine are deemed to be the Heyland diagram of the $p$ pole pair machine and torque/speed curve. For the development of this type of model, it is helpful to redefine the relevant space vector equations in terms of phasors. The general relationship in steady-state between a space vector $\vec{x}$ and phasor $\bar{x}$ is of the form $\vec{x} = \bar{x}e^{j\omega_s t}$ [68]. Given this relationship, the supply voltage phasor may be expressed as $\bar{u}_s = \hat{u}_s$. Furthermore, the phasor representation of the rotor flux linkage on the stator and rotor side of the IRTF may be written as
where \( \theta \) is equal to \( \omega_m t \) (constant speed operation). Use of (8.19) and (8.28) leads to the following phasor based equation set for the machine

\[
\begin{align*}
\ddot{\psi}_R &= \dot{\psi}_R e^{j\omega_m t} \\
\ddot{\psi}_{xy} &= \dot{\psi}_{xy} e^{j(\omega_m t - \theta)}
\end{align*}
\] (8.28a, 8.28b)

Elimination of the flux linkage phasor from (8.29b) and (8.29c) leads to an expression for the air-gap EMF on the stator and rotor side of the IRTF namely

\[
\ddot{e}_{xy} = \ddot{e}_R s
\] (8.30)

where \( s \) is known as the slip of the machine and is given by

\[
s = 1 - \frac{\omega_m}{\omega_s}. \quad (8.31)
\]

The slip according to (8.31) is simply the ratio between the rotor rotational frequency \( \omega_{sl} = \omega_s - \omega_m \) (as apparent on the rotor side of the IRTF) and the stator rotational frequency \( \omega_s \). The Heyland diagram for this model is found by making use of (8.29) which leads to the following expression for the phasor based stator current:

\[
\dot{i}_s = \frac{\ddot{u}_s (R_s + j\omega_s L_M)}{j\omega_s L_M R_s + (R_s + j\omega_s L_{\sigma S}) (R_R + j\omega_s L_M)}. \quad (8.32)
\]

The equivalent circuit which corresponds with (8.32) is shown in Fig. 8.31.
A normalization of expression (8.32) is convenient, which is of the form \( i_n^s = \frac{i_s}{\hat{u}_s/\omega_s L_{os}} \). An example of an Heyland diagram as calculated using (8.32) (in normalized form) is given in Fig. 8.32(a), for a slip range of \(-1 \leq s \leq 1\) using parameters of the 22 kW machine discussed in the tutorial linked to this part of the book (see Sect. 8.6.8).

![Diagram](image)

Fig. 8.32 Steady-state characteristics of voltage source connected induction machine, complete model according to Fig. 8.31

The corresponding torque \( T_e \) of this machine may be calculated as

\[
T_e = \frac{1}{\omega_s} \left( \Re \left\{ u_s i_s^* \right\} - R_s i_s i_s^* \right)_{\text{p_{air-gap}}}. \tag{8.33}
\]

Equation (8.33) underlines the fact that the torque can be calculated on the basis of the air-gap power \( p_{\text{air-gap}} \), i.e., the power which crosses the air-gap to the rotor (see (8.36)). A normalization of (8.33) (as introduced in [68]) of the form \( T_n = \frac{T_e}{T_e} \), with \( T_{\text{e}} = \frac{u_s^2}{2\omega_s^2 L_{os}} \), leads, with the aid of (8.32), to the torque speed curve given in Fig. 8.32(b).

8.4 Parameter Identification and Estimates for Stator and Rotor Flux Linkage Magnitude

The direct and quadrature stator flux model as given by Fig. 8.27 is also useful in terms of obtaining estimates for the parameters \( L_s, L_{\sigma R}, \) and \( R_R \) on the basis of the measured stator resistance \( R_s \), measured no-load stator
current, and given nameplate data. Furthermore, the approach given here provides a guideline for the flux linkage magnitudes to be used for vector control. The first part of this section outlines the calculation steps which leads to these parameters. Under no-load steady-state conditions, the stator flux oriented model is greatly simplified given that $i_{sq} = 0$ and $\omega_m = \omega_s$. This means that the d-axis and q-axis voltages are given by $u_{sd} = R_s i_{sd}$ and $u_{sq} = \omega_s \psi_s$ respectively. The no-load stator current (vector amplitude) $i_{s_{noload}} = i_{sd}$. Consequently, the stator flux linkage can be calculated according to

$$\psi_s = \frac{1}{\omega_s} \sqrt{u_s^2 - (R_s i_{s_{noload}})^2}$$  \hspace{1cm} (8.34)

where $u_s$ represents the applied rated stator voltage (vector amplitude, power invariant). The self inductance $L_s = L_M$ is, according to Fig. 8.27, of the form

$$L_s = \frac{\psi_s}{i_{s_{noload}}}$$  \hspace{1cm} (8.35)

where $\psi_s$ is calculated using (8.34). An estimate for the rotor resistance $R_R$ is found by considering Fig. 8.27 under steady-state rated torque conditions. Hence, we assume the machine is operating at its nameplate rated speed $\omega_{nom}$ (electrical rated shaft speed in rad/s). The rated electrical torque $T_{e_{nom}}$ is then estimated by calculating the so-called air-gap power $T_e \omega_s$, which is equal to the terminal input power minus the stator copper losses which gives

$$T_{e_{nom}} = \frac{1}{\omega_s} \left( u_s i_{s_{nom}}^{\cos(\rho_s)} - R_s (i_{s_{nom}}^2) \right)$$  \hspace{1cm} (8.36)

where $\omega_s$ represents the electrical stator frequency $2\pi f$, with frequency $f$ in Hz. The variable $i_{s_{nom}}$ represents the rated stator current vector amplitude, while $\rho_s$ represents the angle between the stator voltage vector and stator current vector (this parameter comes directly from the nameplate data, i.e., power factor). On the basis of the rated torque we can calculate the rated quadrature current $i_{s_{sq}}^{nom}$ using (8.34) and (8.36) as

$$i_{s_{sq}}^{nom} = \frac{T_{e_{nom}}}{\psi_s}$$  \hspace{1cm} (8.37)

From the quadrature model, shown in Fig. 8.27, we can derive, under steady-state rated conditions, the voltage equation $e_{sq} = (\omega_s - \omega_m)L_{\sigma R} i_{sd} + R_R i_{sq} + \omega_m \psi_s$, which under the assumption of $L_{\sigma R} i_{sd} \ll \psi_s$ reduces to

$$R_R \simeq \frac{(\omega_s - \omega_m^{nom}) \psi_s}{i_{s_{sq}}^{nom}}$$  \hspace{1cm} (8.38)

The remaining parameter, being the leakage inductance $L_{\sigma R}$, comes from Fig. 8.30(a) (with $a = L_s/L_{\sigma R}$). This figure shows that the direct axis current
value $i_{sd}$ increases as the torque increases. The Heyland diagram circle is a direct function of the leakage. It may be shown that we can approximate the direct current $i_{sd}$ versus $i_{sq}$ function as

$$i_{sd} \simeq \frac{\psi_s}{L_s} + L_{\sigma R} \frac{i_{sq}^2}{\psi_s},$$  \hspace{1cm} (8.39)$$

Equation (8.39), when used under rated conditions, i.e., $i_{sd} = i_{sd}^{\text{nom}}$ and $i_{sq} = i_{sq}^{\text{nom}}$, gives us a direct estimate for the leakage inductance namely

$$L_{\sigma R} \simeq \left( i_{sd}^{\text{nom}} - i_{s}^{\text{no load}} \right) \frac{\psi_s}{i_{sq}^{\text{nom}}},$$  \hspace{1cm} (8.40)$$

where $i_{sd}^{\text{nom}}$ is calculated using the stator current $i_{s}^{\text{nom}}$ and (8.37) as

$$i_{sd}^{\text{nom}} = \sqrt{(i_{s}^{\text{nom}})^2 - (i_{sq}^{\text{nom}})^2}.$$  \hspace{1cm} (8.41)$$

The conversion to a model with $a = 1$ is based on the assumption of equal rotor and stator leakage inductances, i.e., $L_{\sigma s} = L_{\sigma r}$ and $L_s = L_r$. Use of (8.12b), with $a = L_s/L_m$ gives

$$L_m = \sqrt{\frac{L_s^3}{L_{\sigma R} + L_s}}$$  \hspace{1cm} (8.42a)$$

$$L_{\sigma s} = L_s - L_m$$  \hspace{1cm} (8.42b)$$

$$R_r = \left( \frac{L_m}{L_s} \right)^2 R_{R}. $$  \hspace{1cm} (8.42c)$$

The stator flux linkage magnitude $\psi_s$ as given by (8.34) and rated currents $i_{sd}^{\text{nom}}$ and $i_{sq}^{\text{nom}}$ provides the basis for calculating a value for the flux linkage $\psi_M$ and use of (8.16b), namely

$$\psi_{M d} = \psi_s - L_{\sigma S} i_{sd}^{\text{nom}}$$  \hspace{1cm} (8.43a)$$

$$\psi_{M q} = -L_{\sigma S} i_{sq}^{\text{nom}}$$  \hspace{1cm} (8.43b)$$

which leads to the required flux linkage magnitude $\psi_M$ for any given transformation value $a$.

$$\psi_M = \sqrt{\psi_{M d}^2 + \psi_{M q}^2}.$  \hspace{1cm} (8.44)$$

The tutorial at the end of this chapter gives an example of using this approach for calculating the rated flux linkage values and motor parameters for a given machine.
8.5 Single-Phase Induction Machines

In this section, we will consider the use of the IRTF based concept for representing single-phase, squirrel-cage based induction machines. The machine in question is provided with two orthogonal-oriented stator windings referred to as the run winding (main winding) and auxiliary winding respectively as indicated in Fig. 8.33. The number of winding turns per phase may in this case differ, hence the phase resistance of the run winding and auxiliary winding are defined as $R_{aux}$ and $R_{run}$ respectively. Furthermore, a factor $k_{aux}$ is introduced which represents the auxiliary-to-run winding turns ratio. A single-phase sinusoidal supply is assumed which is directly connected to the run winding.

Types of Single-Phase Induction Machines

There are various types of single-phase induction machines. For so-called split-coil machines, the auxiliary winding is also connected directly to the supply source. For capacitor-start type machines, a capacitor $C$ is placed in series with the auxiliary winding and the supply (see Fig. 8.33) and a shaft speed operated switch (also shown in Fig. 8.33) disconnects the auxiliary winding when the speed reaches a predetermined operating speed. So-called capacitor-run machines, first proposed by Steinmetz, are provided with a fixed capacitor between supply and winding which removes the need for a switch. A combination of phase winding topologies described above are found in the industry, hence it is of interest to present a generalized modeling approach in this section which will allow the user to examine the dynamic and steady-state behavior of such machines.

![Fig. 8.33 Generalized wiring diagram of single-phase motor](image)

Dynamic Model of Single-Phase Induction Machine

A convenient starting point for this modeling process is the rotor flux based machine model discussed in Sect. 8.3.2.1. The use of a space vector model is particularly helpful given that this is in fact a two-phase representation of the induction machine. On the stator side of the IRTF components and variables
linked to the \( \alpha \)-axis may, for example, be assigned to the auxiliary winding as is the case in this section. With this choice of axis assignment, the run winding parameters and variables are tied to the \( \beta \)-space vector axis hence the supply voltage and current space vectors \( \vec{u}_{ss} \) and \( \vec{i}_{ss} \) may be written as

\[
\vec{u}_{ss} = u_{aux} + j u_{run} \quad (8.45a)
\]

\[
\vec{i}_{ss} = i_{aux} + j i_{run} \quad (8.45b)
\]

where \( u_{aux}, u_{run}, i_{aux}, \) and \( i_{run} \) are given in Fig. 8.33. For the generalized model assumed here, a capacitor \( C \) is connected in series with the auxiliary winding in which case the winding voltage \( u_{aux} \) may be found using

\[
u_{aux} = u_{ss} - \frac{1}{C} \int i_{aux} \, dt \quad (8.46)
\]

where \( u_{ss} \) represents the supply voltage to the machine which is arbitrarily defined as

\[
u_{ss} = \hat{u}_{ss} \cos \omega_s t. \quad (8.47)
\]

The development of a symbolic model for single-phase induction machines is readily initiated by considering the rotor flux based IRTF model according to Fig. 8.14. The parameters \( L_{\sigma S}, L_M, \) and \( R_R \) used to model standard three-phase machines as discussed in Sect. 8.3.2.1 are based on the stator referred parameters \( L_{\sigma S}, L_{\sigma S}, L_m, \) and \( R_r \). Here, the number of turns on the run and auxiliary winding may differ and a choice needs to be made in terms of referring said parameters to a specific winding. The run winding is assigned as the reference winding and the machine parameters are referred to this winding. The resulting symbolic model for the single-phase machine as given in Fig. 8.34 has resemblance with the rotor flux based machine model for the three-phase machine from Fig. 8.14 which was the starting point of the modeling process.

![Fig. 8.34 IRTF/ITF based symbolic model for single-phase induction machine](image-url)
However, in this case an asymmetric ITF module is introduced in order to accommodate the difference in run/auxiliary winding turns. The latter is achieved by introducing an ITF module where the \( \alpha \)-axis (assigned to the auxiliary winding) winding ratio is set to \( k_{aux} : 1 \), while the \( \beta \)-axis winding ratio is set to unity. The ITF equation set is therefore of the form

\[
\begin{align*}
\dot{i}_{s\alpha} &= k_{aux} i_{aux} \\
\dot{i}_{s\beta} &= i_{run}
\end{align*}
\]

(8.48a) \hspace{1cm} (8.48b)

while the corresponding ITF flux linkages may be defined as

\[
\begin{align*}
\psi_{aux} &= k_{aux} \psi_{s\alpha} \\
\psi_{run} &= \psi_{s\beta}
\end{align*}
\]

(8.49a) \hspace{1cm} (8.49b)

where the variables \( \psi_{aux} \) and \( \psi_{run} \) are introduced which are linked with the space vector \( \vec{\psi}_{ss} = \psi_{aux} + j\psi_{run} \). The equation set which corresponds to Fig. 8.34 is of the form

\[
\begin{align*}
\dot{u}_{aux} &= R_{aux} i_{aux} + \frac{d\psi_{aux}}{dt} \\
\dot{u}_{run} &= R_{run} i_{run} + \frac{d\psi_{run}}{dt} \\
\vec{\psi}_{s} &= \vec{\psi}_{R} + L_{\sigma s} \vec{i}_{s} \\
\frac{\vec{\psi}_{xy}^{R}}{L_{M}} &= \vec{i}_{s}^{xy} - \vec{i}_{R}^{xy} \\
\dot{\vec{e}}_{R}^{xy} &= \frac{d\vec{\psi}_{R}^{xy}}{dt} \\
\dot{e}_{R}^{xy} &= R_{R} \dot{\vec{e}}_{R}^{xy}
\end{align*}
\]

(8.50a) \hspace{1cm} (8.50b) \hspace{1cm} (8.50c) \hspace{1cm} (8.50d) \hspace{1cm} (8.50e) \hspace{1cm} (8.50f)

where the parameters \( R_{aux} \) and \( R_{run} \) represent the resistance of the auxiliary and run winding respectively. Note that the single-phase symbolic model and corresponding equation set reduce to the rotor flux oriented model from Sect. 8.3.2.1 in case the auxiliary and run windings are identical in terms of the winding configuration. Under these circumstances, the phase resistances will be equal and the winding ratio \( k_{aux} \) will be equal to 1, in which case the ITF module can be omitted from the symbolic diagram.

The generic model as given by Fig. 8.35 which corresponds to the symbolic model shown in Fig. 8.34 and equation set (8.50) shows the introduction of scalar variables on the primary side of the (asymmetric) ITF module.

Tutorial Results

In Sect. 8.6.9, a simulation example is given which is based on the model described above. In this tutorial, the model is used to examine the line start of a 150 W four-pole, capacitor-run type single-phase induction machine. For this purpose, the model voltage input \( u_{run} \) is connected to a 50 Hz sinusoidal
Fig. 8.35 Generic model for single-phase induction machines

voltage source as defined by (8.47). Furthermore, the model voltage $u_{aux}$ is derived with the aid of (8.46) which may be implemented with the aid of an additional integrator module and gain module with gain $1/C$. An example of the results obtained with this simulation model, as indicated in Fig. 8.36, shows the machine shaft torque and shaft speed during a 2 s start-up sequence where a load torque step is applied at $t = 1.5$ s. The reader is referred to the tutorial given in Sect. 8.6.9 for further details and the opportunity to interactively examine this model concept.

It is emphasized that the capacitor-run machine model can be readily adapted to different single-phase machine concepts. For the purpose of modeling a capacitor start machine, a speed dependent switch may be introduced which purposely sets the auxiliary winding resistance $R_{aux}$ to a substantially larger (in comparison with the auxiliary winding resistance) value. For the purpose of modeling a split-phase induction machine, the capacitor gain module $1/C$ may be simply set to zero in which case both voltage inputs of the model are connected to the same sinusoidal voltage source. Modeling of, for example, a TRIAC controlled single-phase machine can be undertaken by
8.5.1 Steady-State Analysis of Capacitor-Run Single-Phase Induction Machines

A phasor based analysis of the generalized single-phase model is considered here to determine the steady-state run and auxiliary peak (or RMS) currents, average torque, and torque ripple as function of the shaft speed for capacitor-run single-phase induction machines.

**Capacitor as Integral Part of the Machine**

For the purpose of this analysis, it is helpful to consider the capacitor in series with the auxiliary winding as an integral part of the machine, i.e., instead of...
defining the stator voltage space vector \( \vec{u}_{ss} \) as
\[
\vec{u}_{ss} = u_{aux} + j u_{run}
\]
as in (8.45) we redefine it as
\[
\vec{u}_{ss} = u_{ss\alpha} + j u_{ss\beta}
\]
(8.51)

where \( u_{ss\alpha} \) is now the voltage drop over the capacitor in series with the auxiliary winding while \( u_{ss\beta} \) remains the resulting voltage drop over the run winding, compare Fig. 8.33. Due to the parallel connection to the single-phase supply voltage, both \( u_{ss\alpha} \) and \( u_{ss\beta} \) equal \( u_{ss} \) (with \( u_{ss} \) defined in (8.47)). The orientation of the space vector components with the auxiliary winding in the \( \alpha \)-direction and the run winding in the \( \beta \)-direction remains unchanged.

For \( u_{ss\alpha} \), (8.50a) is substituted into (8.46) which leads to
\[
\begin{align*}
    u_{ss\alpha} &= u_{aux} + \frac{1}{C} \int i_{aux} \, dt \\
    \Leftrightarrow u_{ss\alpha} &= R_{aux} i_{aux} + \frac{1}{C} \int i_{aux} \, dt + \frac{d\psi_{aux}}{dt}
\end{align*}
\]
(8.52)

where \( u_{ss} \) is defined by (8.47). The run winding is directly connected to the supply source and hence \( u_{ss\beta} \) is formed by
\[
\begin{align*}
    u_{ss\beta} &= u_{run} \\
    \Leftrightarrow u_{ss\beta} &= R_{run} i_{run} + \frac{d\psi_{run}}{dt}.
\end{align*}
\]
(8.53)

**Phasor Notation of Voltage Equations**

In steady-state, the auxiliary and run voltages \( u_{ss\alpha} \) and \( u_{ss\beta} \) are sinusoidal functions of time. A sinusoidal quantity is represented by the expression \( u = \hat{u} \cos(\omega_s t + \rho) \) and can be expressed as \( u = \Re\{u e^{j\omega_s t}\} \) where the phasor \( u = \hat{u} e^{j\rho} \) is introduced. In phasor notation and matrix format, (8.52) and (8.53) can therefore be rewritten as
\[
\begin{bmatrix}
    u_{ss\alpha} \\
    u_{ss\beta}
\end{bmatrix}
= \begin{bmatrix}
    Z_{aux} & 0 \\
    0 & R_{run}
\end{bmatrix}
\begin{bmatrix}
    i_{run} \\
    i_{aux}
\end{bmatrix}
+ j \omega_s \begin{bmatrix}
    \psi_{aux} \\
    \psi_{run}
\end{bmatrix}
\]
(8.54)

where the impedance \( Z_{aux} = R_{aux} + \frac{1}{j\omega_s C} \).

Equation set (8.54) is now referred to the secondary side of the ITF module shown in Fig. 8.34. This is achieved by multiplying (8.54) by a factor \( \frac{1}{k_{aux}} \) which gives
\[
\begin{bmatrix}
    u_{ss\alpha} \\
    u_{ss\beta}
\end{bmatrix}
= \begin{bmatrix}
    Z_{s\alpha} & 0 \\
    0 & Z_{s\beta}
\end{bmatrix}
\begin{bmatrix}
    i_{s\alpha} \\
    i_{s\beta}
\end{bmatrix}
+ j \omega_s \begin{bmatrix}
    \psi_{s\alpha} \\
    \psi_{s\beta}
\end{bmatrix}
\]
(8.55)
with

\[ u_{s\alpha} = \frac{u_s}{k_{aux}} \]
\[ u_{s\beta} = u_s \]
\[ i_{s\alpha} = k_{aux} i_{aux} \]
\[ i_{s\beta} = i_{run} \]
\[ \psi_{s\alpha} = \frac{\psi_{aux}}{k_{aux}} \]
\[ \psi_{s\beta} = \psi_{run} \]

and

\[ Z_{s\alpha} = \frac{Z_{aux}}{k_{aux}^2} \]
\[ R_{s\beta} = R_{run} \]

Positive and Negative Sequence Phasors

With the aid of Euler’s formula, the time domain quantities \( u_{s\alpha} \) and \( u_{s\beta} \) can be expressed by their phasor from the above (8.55) as

\[ u_{s\alpha} = \frac{1}{2} \left( u_{s\alpha} e^{j\omega_s t} + u_{s\alpha}^* e^{-j\omega_s t} \right) \quad (8.56a) \]
\[ u_{s\beta} = \frac{1}{2} \left( u_{s\beta} e^{j\omega_s t} + u_{s\beta}^* e^{-j\omega_s t} \right) . \quad (8.56b) \]

An observation of (8.56) shows that the two sinusoidal scalar quantities \( u_{s\alpha} \) and \( u_{s\beta} \) may be represented by two counter-rotating space vectors and two phasors \( u_{s\alpha} \) and \( u_{s\beta} \).

The two time domain quantities \( u_{s\alpha} \) and \( u_{s\beta} \), according to the symbolic and generic diagram of the machine, represent the space vector \( \vec{u}_s = u_{s\alpha} + j u_{s\beta} \). Use of (8.56) with this expression and grouping the phasors linked to the terms \( e^{j\omega_s t} \) and \( e^{-j\omega_s t} \) gives

\[ \vec{u}_s = \left( \frac{1}{2} u_{s\alpha} + \frac{1}{2} u_{s\beta} \right) e^{j\omega_s t} + \left( \frac{1}{2} u_{s\alpha}^* + \frac{1}{2} u_{s\beta}^* \right) e^{-j\omega_s t} . \quad (8.57) \]

The resultant equation as represented by expression (8.58) shows that the space vector \( \vec{u}_s \) may also be represented by two counter-rotating space vectors which in turn are linked to two phasors \( u_{s+} \) and \( u_{s-} \) that are tied to the rotation direction. The subscript notation \(+, -\) underlines the fact that these phasors are referred to as the so-called positive and negative sequence phasors,
\[ \vec{u}_s = u_{s+} e^{j\omega_s t} + u_{s-}^* e^{-j\omega_s t}. \] (8.58)

Observation of (8.57) shows that the relationship between the phasors \( u_{s\alpha} \) and \( u_{s\beta} \) and phasors \( u_{s+} \) and \( u_{s-} \) may be expressed in terms of a matrix based expression (8.59),

\[
\begin{bmatrix}
  u_{s+} \\
  u_{s-}
\end{bmatrix} = \frac{1}{2} \begin{bmatrix}
  1 & j \\
  1 & -j
\end{bmatrix}
A
\begin{bmatrix}
  u_{s\alpha} \\
  u_{s\beta}
\end{bmatrix}
\] (8.59)

where \( A \) is referred to as the transformation matrix. For three-phase machines operating under symmetrical conditions, the two phasors \( u_{s\alpha} \) and \( u_{s\beta} \) will be orthogonal and symmetric. If, for example, the two phasors are set to \( u_{s\alpha} = \hat{u}_s \) and \( u_{s\beta} = -j\hat{u}_s \) respectively, (8.58) is reduced to \( \vec{u}_s = \hat{u}_s e^{j\omega_s t} \), i.e., the negative sequence phasor \( u_{s-} \) will then be zero.

The positive and negative phasor variables may be transformed to the \( \alpha \) and \( \beta \)-phasor as

\[
\begin{bmatrix}
  u_{s\alpha} \\
  u_{s\beta}
\end{bmatrix} = \begin{bmatrix}
  1 & 1 \\
  -j & j
\end{bmatrix}
A^{-1}
\begin{bmatrix}
  u_{s+} \\
  u_{s-}
\end{bmatrix}.
\] (8.60)

This transform is the inverse of (8.59). The matrix \( A^{-1} \) is the inverse matrix of the matrix \( A \).

Positive and Negative Sequence Model

The transformation process described above for the two scalar variables \( u_{s\alpha} \) and \( u_{s\beta} \) can be equally applied to other machine variables such as the flux linkage and current. Consequently, the conversion process in question may be used to develop a so-called positive and negative sequence phasor model of the machine, with inputs \( u_{s+} \) and \( u_{s-} \).

The transformation to positive and negative sequence based phasors is achieved by multiplying both sides of (8.55) with the transformation matrix \( A \) as given in (8.59). The resulting equation is

\[
\begin{bmatrix}
  u_{s+} \\
  u_{s-}
\end{bmatrix} = \frac{1}{2} \begin{bmatrix}
  Z_{s\alpha} + R_{s\beta} & Z_{s\alpha} - R_{s\beta} \\
  Z_{s\alpha} - R_{s\beta} & Z_{s\alpha} + R_{s\beta}
\end{bmatrix}
\begin{bmatrix}
  i_{s+} \\
  i_{s-}
\end{bmatrix} + j\omega_s \begin{bmatrix}
  \psi_{s+} \\
  \psi_{s-}
\end{bmatrix}.
\] (8.61)

In this expression, the terms \( \psi_{s+} \) and \( \psi_{s-} \) need to be considered in detail. This is achieved by rewriting (8.50c) in terms of positive and negative sequence phasors. The resultant equation set, which has been multiplied by factor \( j\omega_s \) to simplify the ensuing analysis, becomes
\[ j\omega_s \psi_{s+} = j\omega_s L \sigma i_{s+} + j\omega_s \psi_{R+} \tag{8.62a} \]
\[ j\omega_s \psi_{s-} = j\omega_s L \sigma i_{s-} + j\omega_s \psi_{R-} \tag{8.62b} \]

where the phasors \( j\omega_s \psi_{R+} \) and \( j\omega_s \psi_{R-} \) appear. These may be further developed by making use of expression (8.50e) in which the space vectors are shown in rotating coordinates. For the conversion to positive and negative sequence phasors using the approach outlined above, it is helpful to reconsider the relationship between stationary and rotating space vectors, namely \( \vec{A} = \vec{A}_r e^{j\theta} \) where \( \theta \) represents the shaft angle which may also be written as \( \theta = \omega_m t \). The positive/negative phasors linked to expression (8.50e) may after some mathematical handling be written as

\[ e_{R+} = \left(1 - \frac{\omega_m}{\omega_s}\right) j\omega_s \psi_{R+} \tag{8.63a} \]
\[ e_{R-} = \left(1 + \frac{\omega_m}{\omega_s}\right) j\omega_s \psi_{R-} \tag{8.63b} \]

Some simplification of equation set (8.63) may be realized by introducing the slip \( s \) as defined by expression (8.31). Note that the slip relationship was defined for a unidirectional set of space vectors. In the case here, two counter rotating space vectors are used, hence we will arbitrarily assume that the slip is determined with respect to the positive rotating vectors. The left-hand side of equation set (8.63) may also be developed further with the aid of (8.50c), (8.50d), and (8.50f) and converting the latter to positive/negative phasors. Subsequent manipulation of (8.63) using the approach outlined and substitution of the phasor based equation into (8.62) gives

\[ j\omega_s \psi_{s+} = i_{s+} \left( j\omega_s L \sigma s + \frac{j\omega_s L M}{R_s} \frac{R_R}{s} + j\omega_s L M \right) \tag{8.64a} \]
\[ j\omega_s \psi_{s-} = i_{s-} \left( j\omega_s L \sigma s + \frac{j\omega_s L M}{R_s} \frac{R_R}{2-s} + j\omega_s L M \right) \tag{8.64b} \]

where \( Z_{s+} \) and \( Z_{s-} \) represent a positive and negative sequence impedance network which consists of the leakage inductance in series with a parallel network formed by a slip-dependent rotor resistance and magnetizing inductance. Note that the positive sequence impedance \( Z_{s+} \) is in fact the network shown in Fig. 8.31 for the steady-state of the three-phase induction machine(without the resistance \( R_s \)), while the negative sequence is similar but utilizes a different slip-dependant rotor resistance.

The positive and negative sequence currents are obtained by substituting the flux linkage expression (8.64) into the voltage (8.61),
\[
\frac{u_{s+}}{u_{s-}} = \frac{1}{2} \left[ \frac{Z_{s\alpha} + R_{s\beta} + 2Z_{s+}}{Z_{s\alpha} - R_{s\beta}} - \frac{Z_{s\alpha} - R_{s\beta}}{Z_{s\alpha} + R_{s\beta} + 2Z_{s-}} \right] \left[ \frac{i_{s+}}{i_{s-}} \right].
\] (8.65)

Hence, on the basis of a given steady-state sinusoidal voltage excitation \(u_s\), the voltage phasors \(u_{s+}\) and \(u_{s-}\) may be found using (8.57). In turn, these phasors are used as an input for expression (8.65) from which the currents \(i_{s+}\) and \(i_{s-}\) for a given shaft speed may be derived. These current phasors can with the aid of (8.60) (for currents instead of voltages) be used to obtain the run and auxiliary currents (in phasor form). For the purpose of developing a steady-state equivalent model it is helpful to rewrite (8.65) as

\[
\begin{align*}
    u_{s+} &= (R_{\text{run}} + Z_{s+}) i_{s+} \\
    &+ \left\{ \frac{1}{2} \left( \frac{R_{\text{aux}}}{k_{\text{aux}}^2} - R_{\text{run}} \right) + \frac{1}{j2\omega_s C k_{\text{aux}}^2} \right\} \left( i_{s+} + i_{s-} \right) \quad \text{(8.66a)} \\
    u_{s-} &= (R_{\text{run}} + Z_{s-}) i_{s-} \\
    &+ \left\{ \frac{1}{2} \left( \frac{R_{\text{aux}}}{k_{\text{aux}}^2} - R_{\text{run}} \right) + \frac{1}{j2\omega_s C k_{\text{aux}}^2} \right\} \left( i_{s+} + i_{s-} \right) \quad \text{(8.66b)}
\end{align*}
\]

where the auxiliary and run winding variables are reintroduced as defined for the dynamic model in order to enhance the readability of the resultant expression. A model representation as introduced in Fig. 8.37 is consistent with mathematical expression (8.66). Readily apparent in this model are the two impedances \(Z_{s+}\) and \(Z_{s-}\) as defined in (8.64).

Fig. 8.37 Equivalent steady-state circuit of a single-phase induction machine with auxiliary capacitor
Torque Calculation in Steady-State

We will now consider the calculation of the average torque and peak amplitude of the pulsating torque under steady-state conditions using the model given in Fig. 8.37. A suitable starting point for this analysis is the general torque expression in phasor notation

\[ T_e = \Im \{ \psi_s^* i_s \}. \]  

(8.67)

This expression may be rewritten in terms of the positive and negative sequence flux linkage and current phasors \( \psi_{s\pm} \) and \( i_{s\pm} \) by making use of (8.58) in which the voltage variables must be replaced by the appropriate space vectors and phasors. Subsequent mathematical manipulation using this approach gives

\[
T_e = \Im \left\{ \psi_{s+}^* i_{s+} \right\} + \Im \left\{ \psi_{s-}^* i_{s-}^* \right\} + \Im \left\{ \psi_{s-}^* i_{s+} e^{j2\omega_s t} \right\} + \Im \left\{ \psi_{s+}^* i_{s-} e^{-j2\omega_s t} \right\}.
\]

(8.68)

Observation of (8.68) shows that the torque expression has an average (non-time-dependent) component \( T_{eav} \) that may be found with the aid of the positive and negative current phasors. Equation (8.68) also shows a sinusoidal torque ripple component \( T_{eripple} \) which has a frequency that is double the voltage supply excitation frequency \( \omega_s \). The torque \( T_e \) can be rewritten as

\[
T_e = T_{eav} + T_{eR} \sin (2\omega_s t) + T_{eX} \cos (2\omega_s t).
\]

(8.69)

with

\[
T_{eR} = \Re \left\{ \psi_{s-}^* i_{s+} \right\} - \Re \left\{ \psi_{s+}^* i_{s-}^* \right\}
\]

\[
T_{eX} = \Im \left\{ \psi_{s-}^* i_{s+} \right\} + \Im \left\{ \psi_{s+}^* i_{s-}^* \right\}.
\]

The peak amplitude \( \hat{T}_{eripple} \) of this torque ripple component is

\[
\hat{T}_{eripple} = \sqrt{(T_{eR})^2 + (T_{eX})^2}.
\]

(8.70)

Tutorial Results

A numerical example of a steady-state analysis for the single-phase capacitor run induction machine used for the dynamic model example is given in Fig. 8.38. The results represent the average torque, peak torque ripple, peak auxiliary, and peak run currents as function of the ratio between shaft speed and synchronous speed \( p\omega_s \) over a slip range from \( s = 0 \) to \( s = 2 \).
Fig. 8.38 Steady-state analysis example of a capacitor-run single-phase induction machine
In this tutorial, a simulation is considered which is based on the generic model as shown in Fig. 8.3. A rotating flux vector $\vec{\psi}_m$ with angular frequency $\omega_s = 100\pi \text{ rad/s}$ and amplitude $\psi_m = 1.25 \text{ Wb}$ is assumed as an input to this model. This means that the integrator with output $\dot{\psi}_m$ (see Fig. 8.3) can be omitted in this tutorial. Use of a flux vector as an input emphasizes the fact that it is flux which plays a key role with respect to the operation of the machine. Furthermore, the use of constant flux vector allows the introduction of an alternative differentiation module [68], which requires a constant amplitude input vector, on the rotor side of the IRTF module. This approach avoids the use of standard differentiator modules which may cause simulation problems. The generation of the vector $\vec{\psi}_m$ is realized with the aid of a polar to Cartesian conversion module which has as input the required vector amplitude $\psi_m$ and angle $\omega_m t$. The machine is connected to a mechanical load with a quadratic torque speed curve, where we are able to vary the load torque value (so we can achieve variable speed motor operation).

**Fig. 8.39** Simulation of induction machine using the simplified model
The aim of this example is to examine the steady-state operation, hence the use of modules which represent key RMS values and real/reactive power modules. The machine in question has the following parameters: inertia of \( J = 0.001 \text{ kg m}^2 \), stator resistance \( R_s = 6.9 \Omega \), rotor resistance \( R_r = 3.0 \Omega \) and magnetizing inductance \( L_m = 340.9 \text{ mH} \). The simulation model as given in Fig. 8.39 satisfies the requirements for this tutorial. It is instructive to activate the CASPOC feature *animate nodal voltages* prior to running the simulation, as this allows the user to visualize the numerical value at named nodes. Space vectors also show a value, namely the amplitude. Furthermore, the simulation should be set to run continuously, by way of the *hand* button on the CASPOC toolbar. In this simulation example the supply voltage is calculated using (8.1a).

### 8.6.2 Tutorial 2: Universal Induction Machine Model

This tutorial is concerned with the development of a simulation model that represents a universal IRTF based machine concept. The aim is to develop a model which reflects the generic model given in Fig. 8.13. Furthermore, the user should be able to vary the transformation variable \( a \) so that the impact of this transformation can be seen on a vector diagram of the type given in Fig. 8.11(b). To achieve this objective the flux vectors \( \vec{\psi}_s, \vec{\psi}_M, \vec{\psi}_R \), should be transformed to a coordinate reference frame that is tied to the flux vector \( \vec{\psi}_m \) (as given in Fig. 8.11(a)). For this tutorial, the model as developed in the previous section is to extended with the following set of inductance parameters: stator inductance \( L_s = 346.9 \text{ mH} \) and rotor inductance \( L_r = 346.9 \text{ mH} \). The remaining set of parameters are as given in the previous tutorial. With this choice of parameters the limits of the transformation variable \( a \) are (see (8.13)) as follows \( 0.9827 \leq a \leq 1.0177 \).

An implementation example as given in Fig. 8.40 shown the presence of the inductance matrix \( L^{-1} \) together with an *UPDOWN* module which allows the user to vary the transformation variable \( a \). As with the previous example, a rotating flux vector is assumed, which in this example represents the stator flux vector \( \vec{\psi}_s \). Furthermore, the gain module \( R_R \) have been replaced by a new variable gain module which has a second input representing the transformation variable \( a \). The gain for this new module is equal to \( R_R = a^2 R_r \) (see (8.15)).

A set of three coordinate transformation modules is introduced in Fig. 8.40 which transform the flux vectors \( \vec{\psi}_s, \vec{\psi}_M, \vec{\psi}_R \), to a synchronous reference frame linked to the vector \( \vec{\psi}_m \). This vector is vector is calculated with the aid of (8.5b), with \( L_{\sigma s} = L_s - L_m \). Furthermore, the instantaneous angle \( \rho_m \) of this vector, as found with the aid of a Cartesian to polar conversion unit, is connected to said coordinate conversion modules to realize the required representation of the flux vectors. When the simulation is active, the user
Fig. 8.40  Simulation of induction machine, universal model

Fig. 8.41  Simulation results of induction machine, universal model
is able to change the transformation variable and observe the effects on the flux vectors and the remaining nodes of the simulation, as discussed in the previous tutorial. Note in particular that the values for the (steady state) output torque/shaft speed and RMS stator voltage/current are unaffected by the changes to the parameter \( a \). Finally, it is instructive to vary the load torque setting and observe the changes to the flux vectors and node values of the simulation model.

### 8.6.3 Tutorial 3: Universal Stationary Oriented Induction Machine Model

This tutorial is concerned with the implementation of a stationary oriented model of the induction machine as discussed in Sect. 8.3.3. The aim is to develop a simulation model which is based on the generic structure given in Fig. 8.19. The machine is to be connected to a three-phase 220 V, 50 Hz sinusoidal source. The machine inductances and resistances are according to those specified in the previous two tutorials. A combined machine/load inertia of \( J = 0.01 \text{ kg m}^2 \) will be used in this simulation. Furthermore, a quadratic load torque versus shaft speed characteristic is assumed. Of interest is to examine the model under steady state conditions by viewing the shaft torque and shaft speed for a given load torque value and to examine the effect on these variables by changing the transformation factor \( a \).

![Simulation of universal model with stationary oriented reference frame](image-url)
The simulation model as given in Fig. 8.42 satisfies the requirements for this tutorial. Excitation for the model is provided by making use of a rotating voltage supply vector $\vec{u}_s$, the amplitude of which is set to $U_s \sqrt{3}$, where $U_s = 220$ V represents the RMS phase voltage value. The inclusion of the factor $\sqrt{3}$ follows from the fact that power invariant space vectors are used throughout this book. Also shown in this figure are the inverse matrix element $L^{-1}$ and gain unit $R_R$ both of which are affected by changes to the transformation variable $a$. If the simulation is executed for different transformation values $a$, in this case realized by using the UPDOWN a value, the user may ascertain that the torque and speed readings remain unaffected. Changes to the load torque setting, realized by way of the load module torque setting, give the user the ability to examine, for example, the steady state torque speed curve of this machine.

8.6.4 Tutorial 4: Current Controlled Zero Leakage Flux Oriented Machine Model

This tutorial is concerned with the development of a zero leakage, UFO based model as discussed in Sect. 8.2.2. A simulation model is envisaged, which complies with the generic structure shown in Fig. 8.6. The machine inductance and resistance parameters are according to those used in the previous tutorial. Furthermore, the machine, which has an inertia of $J = 0.1$ kg$\cdot$m$^2$, is deemed to operate under no-load conditions. A direct and quadrature current are to be used as inputs, where the direct axis is to be set to $i_{dl} = 0$ A at the start of the simulation. A step in the quadrature current from $0 \rightarrow 4$ A is required, at $t = 0.8$ s. The aim is to build a model of the machine and examine the transient response in terms of the shaft torque $T_e$, flux linkage $\psi_m$ and speed variables $\omega_m$, $\omega_{sl}$, $\omega_s$. An implementation example which satisfies the requirements for this tutorial, as shown in Fig. 8.43, was used to derive the result shown in Fig. 8.7. A simulation step time of 100 $\mu$s and run time of 2.0 s has been assumed to give the reader a clear visualization of the machine torque, flux linkage and speed response over time. It is helpful to active the cross hairs/red dot button in the CASPOC tool bar, which activates the node animation feature of the program. This allows the reader access to the numerical values of labeled nodes during the simulation. This feature in combination with the replay feature (activated with the red dot button on the bottom tool bar) allows the user to examine the node variables at any instant of the simulation sequence. The scopes used to present the results are shown in Fig. 8.44.
Fig. 8.43  Simulation of current controlled, flux oriented model with zero leakage

Fig. 8.44  Simulation results of current controlled, flux oriented model with zero leakage
8.6.5 Tutorial 5: Current Controlled Universal Field Oriented (UFO) Model

In this tutorial a generalized UFO model, with leakage inductances, is to be developed which is based on the generic structure shown in Fig. 8.22. The aim is to consider the transient response of the model in question, whereby the user is able to alter the transformation ratio $a$. For this purpose the current excitation and machine inertia as given in the previous tutorial are again used in this example. Furthermore, the machine resistance and inductance parameters for this tutorial are given in Sect. 8.6.2. The simulation model, as shown in Fig. 8.45, complies with the generic model of the machine in its present form (see Fig. 8.22). Setting the transformation variable $a$ to either side of its scale using the UPDOWN module leads to a rotor or stator flux oriented model, with corresponding transient results given by Fig. 8.26 and Fig. 8.29 respectively. The scopes used to present the results are similar to Fig. 8.44.

Fig. 8.45 Simulation of current controlled, flux oriented model with leakage inductance
8.6.6 Tutorial 6: Parameter Estimation Using Name Plate Data and Known Stator Resistance

The name plate data given in Table 8.1 corresponds to a delta connected motor, of which the measured RMS no-load line current and resistance between two terminals (with the machine at standstill) were found to be equal to $I_{\text{inload}} = 8.4$ A and $R_1 = 0.35\,\Omega$ respectively. On the basis of the approach set out in Sect. 8.4 build a MATLAB m-file which calculates the parameters $L_s, L_{\sigma s}, L_{\sigma r}$, and $R_r$, whereby it may be assumed that the leakage inductances are equal. Furthermore, calculate the rated stator flux $\vec{\psi}_s$, magnetizing flux $\psi_M$ and the parameters $L_{\sigma S}, L_M, R_R$ for a rotor flux field oriented model.

<table>
<thead>
<tr>
<th>Table 8.1 Name plate data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameters</td>
</tr>
<tr>
<td>Rated power</td>
</tr>
<tr>
<td>Rated shaft speed</td>
</tr>
<tr>
<td>Number of pole pairs</td>
</tr>
<tr>
<td>Rated RMS line voltage</td>
</tr>
<tr>
<td>Rated RMS line current</td>
</tr>
<tr>
<td>Supply frequency</td>
</tr>
<tr>
<td>Rated power factor</td>
</tr>
</tbody>
</table>

A possible approach to this problem may be initiated by examining the measured resistance $R_1$ between two terminals of the machine. For a delta connected machine, the equivalent resistance seen by two terminals is equal to resistance of one phase with the other two series connected phases in parallel. Hence, the stator resistance is equal to $R_s = 3/2R_1$. The RMS no-load phase current and RMS rated phase current are equal to $i_{\text{inload}}^D = I_{\text{inload}}/\sqrt{3}$ and $i_{\text{D}}^{\text{nom}} = I_{\text{nom}}/\sqrt{3}$ respectively, given the use of a delta connected machine. Furthermore, the conversion from RMS phase variables to power invariant space vector amplitude is realized by multiplying the former with a factor $\sqrt{3}$ [68]. Hence, the magnitude of the current and voltage space vectors defined in (8.34) are equal to $u_s = U_{\text{nom}}^s/\sqrt{3}, i_{\text{inload}}^s = I_{\text{inload}}$, which allows the calculation of the stator flux linkage vector amplitude $\vec{\psi}_s$ and inductance $L_s$ given that $\omega_s = 2\pi f_s$. Calculation of the rated electromechanical torque proceeds with the aid of (8.36) which requires access to the rated (space vector) current amplitude $i_{\text{nom}}^s = I_{\text{nom}}^s$ (note that the amplitude of a power invariant current space vector is equal to the RMS line current value, in case the machine is delta connected). The remaining variables required for this calculation are found with the aid of Table 8.1. Computation of the parameters $L_s, L_{\sigma s}, L_{\sigma r}, R_r$, with the assumption of $L_{\sigma s} = L_{\sigma r}$, proceeds using the approach set out in Sect. 8.4 which leads to the data shown in Table 8.2. Also included (for completeness) in this table is the measured stator resistance and homopolar inductance $L_{\text{hom}}$. 
Table 8.2 Estimated machine parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stator inductance</td>
<td>$L_s$</td>
</tr>
<tr>
<td>Magnetizing inductance</td>
<td>$L_m$</td>
</tr>
<tr>
<td>Leakage inductance</td>
<td>$L_{gs}$</td>
</tr>
<tr>
<td>Leakage inductance</td>
<td>$L_{gt}$</td>
</tr>
<tr>
<td>Rotor resistance</td>
<td>$R_r$</td>
</tr>
<tr>
<td>Stator resistance</td>
<td>$R_s$</td>
</tr>
<tr>
<td>Homopolar inductance</td>
<td>$L_{hom}$</td>
</tr>
</tbody>
</table>

The second part of this tutorial requires the conversion of the parameters given in Table 8.2 to a revised set of universal model parameters as defined by (8.12) and (8.15) with $a = L_m/L_r$. The new set of parameters as given in Table 8.3 correspond to a rotor flux oriented UFO model which in turn are used in conjunction with (8.43) and (8.44) to calculate the required (rated) magnetizing flux amplitude $\psi_{nom}^M = \psi_{nom}^R$. This flux linkage value together with the earlier found stator flux linkage value and measured stator resistance are also shown in Table 8.3.

Table 8.3 Estimated machine parameters and flux linkage values for a rotor flux oriented model

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnetizing inductance</td>
<td>$L_M$</td>
</tr>
<tr>
<td>Leakage inductance</td>
<td>$L_{gs}$</td>
</tr>
<tr>
<td>Leakage inductance</td>
<td>$L_{gt}$</td>
</tr>
<tr>
<td>Rotor resistance</td>
<td>$R_R$</td>
</tr>
<tr>
<td>Stator resistance</td>
<td>$R_s$</td>
</tr>
<tr>
<td>Rated stator flux linkage</td>
<td>$\psi_{nom}^s$</td>
</tr>
<tr>
<td>Rated rotor flux linkage</td>
<td>$\psi_{nom}^R$</td>
</tr>
</tbody>
</table>

M-File Code:

```matlab
% 22kW machine, delta connected
VsR = 415;  % RMS line voltage in V
IsR = 33.4;  % rated RMS line current in A
IsN = 8.4;   % no load RMS line current in A
Rlin = 0.35; % measured line-to-line resistance in Ohms
% nameplate data
PoutR = 22000; % rated output power in W
pfR = 0.88;   % rated power factor
p = 2;        % four pole machine
nR = 1465;    % rated shaft speed in r/min
fs = 50;      % supply frequency in Hz
%
ws = 2*pi*fs; % electrical freq in rad/sec
wm = p*2*pi*nR/60; % rated electrical shaft freq in rad/sec
Rs = 3/2*Rlin; % stator resistance in Ohms
isR = sqrt(3)*IsR/sqrt(3); % power invariant vector is value
```
isN = sqrt(3)*IsN/sqrt(3); % power invariant no load is value
usR = sqrt(3)*VsR; % power invariant supply voltage vector
Te = (usR*isR*pfR-isR^2*Rs)/ws; % rated electrical torque in Nm
% calculate remaining data
psi_s = 1/ws*sqrt(usR^2-isN^2*Rs); % stator flux amplitude in Vs
Ls = psi_s/isN; % self inductance stator in H
isqR = Te/psi_s; % rated isq value in A
isdR = sqrt(isR^2-isqR^2); % rated isd value in A
RR = (ws-wm)*psi_s/isqR; % rotor resistance with a=Ls/Lm
Lrsig = (isdR-isN)/(isqR^2/psi_s); % leakage inductance with a=Ls/Lm
% calculate parameters for a=1 assume Lsig=r=Lsig
Lr = Ls; % rotor inductance equal to stator inductance
Lm = sqrt(Ls^3/(Lrsig+Ls)); % magnetizing inductance in H
Lsign = Ls-Lm; % stator leakage inductance in H
Rr = (Lm/Ls)^2*RR; % rotor resistance in Ohms
% parameters and flux for any 'a': example rotor flux orientation
a = Lm/Lr;
Lssig = Lm*(Ls/Lm-a);
L_M = a*Lm;
R_R = a^2*Rr;
psiMd = psi_s-Lssig*isdR;
psiMq = -Lssig*isqR;
psi_M = sqrt(psiMd^2+psiMq^2); % rated magnetizing flux in Vs

8.6.7 Tutorial 7: Grid Connected Induction Machine

A dynamic model of a three-phase delta connected induction machine, with an inertia of \( J = 0.1 \text{kg} \cdot \text{m}^2 \) is to be considered which is to be connected to a 415 V (RMS line to line voltage) 50 Hz supply. A four parameter, two pole pair machine representation is envisaged, with a stator oriented leakage inductance and a set of parameters as given in Table 8.3. Provide a simulation model of this machine with the required supply source and plot the shaft torque \( T_e \), line current \( i_l \), stator flux amplitude \( |\vec{\psi}_s| \) and shaft speed \( \omega_m \) over a 2s period. The machine is initially assumed to operate without a mechanical load. However, at time mark \( t = 1 \text{s} \) a 120 Nm (rated) torque step is to be applied. Furthermore, add a set of modules to your simulation which provide a numerical indication of the input power, RMS line current \( I_L \) and RMS phase voltage \( U_D \).

A possible implementation of this problem may be realized by making use of the generic machine model shown in Fig. 8.15, where the IRTF module needs to be extended by a pole pair module in [68]. The introduction of this pole pair module is readily apparent in the model for this tutorial shown in Fig. 8.46. Shown in this simulation model is a space vector generator which has as output a power invariant rotating (at a speed of 3000 rpm) space vector with an amplitude of 415 V. Note that for power invariant space vectors the amplitude is equal to the RMS value of the three individual phases times a factor \( \sqrt{3} \). A set of conversion modules is introduced which realizes the
star–delta conversion \( \vec{u}_{\text{supply}} \rightarrow \vec{u}_{s} \), where \( \vec{u}_{s} \) is the input voltage space vector for the machine model. Conversely, a similar set of modules is introduced in order to realize the delta-star conversion process for the stator current vector \( \vec{i}_{s} \). Also shown in Fig. 8.46 are a set of modules which generate the RMS supply current \( I_{l} \), RMS phase voltage \( U_{D} \) and input power \( p_{in} \).
8.6.8 Tutorial 8: Steady State Characteristics, Grid Connected Induction Machine

This tutorial is concerned with the computation of the steady-state characteristics: $T_e$, $I_l$, $\bar{\psi}_s$, $P_{out}$ of the delta connected machine discussed in the previous subsection. Provide a MATLAB m-file which contains a phasor based analysis for the machine in question when connected to a three phase sinusoidal supply as used in the previous example. Assume the machine input voltage phasor $\bar{u}_s$ as real and calculate the corresponding stator current phasor and torque of the machine using the approach set out in Sect. 8.3.6. Display your results by way of a set of subplots which represent (as function of the shaft speed) the mechanical torque $T_e$, RMS line current $I_l$, flux linkage $|\bar{\psi}_s|$ and output power $P_{out}$. Choose the shaft speed range for your plots so that they coincide with a slip variation of $s : 1 \to 0$. 

Fig. 8.47 Simulation results of grid connected induction machine
A possible solution to this problem may be found by considering the equivalent circuit of the machine given in Fig. 8.31. The model in question utilizes a stator based leakage inductance, hence the machine parameters must be converted as discussed in the tutorial given in Sect. 8.6.6. The machine is delta connected, hence the input voltage phasor is of the form $u_s = 415 \sqrt{3}$, which in turn may be used in conjunction with (8.32) to find the stator current phasor $i_s$. For a delta connected machine, the RMS line current magnitude is equal to $i_s$, as calculated above. Computation of the torque may be realized with the aid of for example (8.33) which must be converted to its phasor equivalent format. This expression requires access to the flux phasor $\psi_R$, which may be found with the aid of (8.29b).

**8.6.9 Tutorial 9: Grid Connected Single-Phase Induction Machine**

This tutorial is concerned with the implementation of a dynamic simulation model of a single-phase, capacitor run induction machine connected to a 110 V(RMS), 50 Hz supply. For this purpose a machine prototype with pa-
rameters as given in Table 8.4 will be considered in terms of its dynamic behavior when subjected to line-start conditions. Of interest is to examine the instantaneous torque, shaft speed and phase currents during the run up sequence of the motor. Central to the approach given here are the concepts discussed in Sect. 8.5 which, as may be observed, provides a more general approach to the topic of modeling single-phase machines. This tutorial is restricted to a capacitor run machine, but the model may be readily adapted to split-coil and capacitor start type machines. In the latter case, the auxiliary winding resistance must be controlled in a manner outlined in the previous tutorial.

Table 8.4 Single-phase, capacitor run machine parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leakage inductance ( L_{\sigma s} )</td>
<td>12.8 mH</td>
</tr>
<tr>
<td>Stator resistance (run winding) ( R_{\text{run}} )</td>
<td>2.02 Ω</td>
</tr>
<tr>
<td>Stator resistance (aux winding) ( R_{\text{aux}} )</td>
<td>7.14 Ω</td>
</tr>
<tr>
<td>Rotor resistance ( R_R )</td>
<td>3.87 Ω</td>
</tr>
<tr>
<td>Magnetizing inductance ( L_M )</td>
<td>171.8 mH</td>
</tr>
<tr>
<td>Asymmetric ITF winding ratio ( k_{\text{aux}} )</td>
<td>1.18</td>
</tr>
<tr>
<td>Capacitor size ( C )</td>
<td>31.0 μF</td>
</tr>
<tr>
<td>Inertia ( J )</td>
<td>1.46 ( 10^{-2} ) kg m²</td>
</tr>
<tr>
<td>Pole pairs ( p )</td>
<td>2</td>
</tr>
<tr>
<td>Initial rotor speed ( \omega_{m0} )</td>
<td>0 rad/sec</td>
</tr>
</tbody>
</table>

A suitable starting point for the proposed analysis is the generic model of the single-phase machine as shown in Fig. 8.35 in which the auxiliary winding must (in this case) be connected to the run winding via a capacitor \( C \). Given this approach, an additional set of generic modules must be provided to implement (8.46). The model as given in Fig. 8.49 shows the ITF and IRTF modules as present in the generic model. In this case the IRTF module is extend with a additional gain module \( p \) given that a four pole machine is used for this tutorial. Also shown is a source module, which is used to apply a 1 Nm load torque step to the machine at \( t = 1.5 \) s.

For the purpose of understanding the torque production mechanism of this machine, it is instructive to consider the locus of the space vectors \( \vec{i}_{sxy} \), \( \vec{\psi}_{Rxy} \) during the simulation sequence. A set of scopes is used to generate the results and these are presented with the aid of MATLAB based subplots as shown in Fig. 8.50. These results clearly show that the presence of a large pulsating torque component, in addition to an average torque component, which (under steady-state conditions) corresponds to the \( (1.0 \text{ Nm}) \) applied load torque value. It is also instructive to examine the interaction between the rotor flux \( \vec{\psi}_{R} \) and stator current vector \( \vec{i}_{s} \) (in rotor coordinates) of the machine, given that these define the instantaneous torque in the machine. The vector plot as given in Fig. 8.51 shows that the relationship between flux and current vectors is by no means constant over the course of the simulation.
This points to the presence of a large torque ripple component as apparent in the result shown on SCOPE3a (see Fig. 8.50).
Fig. 8.50 Simulation results of a line start of single-phase capacitor run induction machine
Fig. 8.51 Line start of single-phase capacitor run induction machine, vector plot $\vec{\psi}_R$ (green trace) and $\vec{i}_{s}/50$ (blue trace)